ASTR 425/525 Cosmology
Dark Matter

(Dated: November 19, 2025)

I. DARK MATTER IN COSMOLOGY

So far in this course, we have encountered dark matter while discussing the energy content of the Universe. We argued that normal baryonic matter forms about 5% of the critical density of the Universe today $\Omega_\mathrm{b} \simeq 0.05$. We also argued that non-relativistic matter as a whole forms about 30% of the critical density of the Universe, $\Omega_\mathrm{m} \simeq 0.3$. The difference between Ω_m and Ω_b is puzzling since in the Standard Model of particle physics there is nothing else that can contribute to the matter density. So, we are forced to postulate that there is something else besides baryons that can contribute to the matter density of our Universe. We call this stuff *dark matter*. From the cosmological perspective, dark matter needs to have the following properties:

- Dark matter behaves like non-relativistic matter; that is, it is pressureless ($w = 0$).
- Dark matter interacts very weakly (if at all) with other particles in the Standard Model.
- Dark matter is essentially collisionless; that is, dark matter particles cannot efficiently exchange energy and momentum among themselves.

The three properties listed above form what we call the *cold dark matter* (CDM) paradigm. CDM as a phenomenological model with $\Omega_\mathrm{c} \simeq 0.25$ has been extremely successful at explaining cosmological observations on large scales ($r > 1$ Mpc), including the cosmic microwave background and the large-scale distribution of galaxies throughout the Universe. But dark matter is also relevant for much smaller scales, including that of galaxies and galaxy clusters. The fact that dark matter shows up pretty much everywhere we look in the Universe reinforces our believe that it must exist.

II. DARK MATTER WITHIN GALAXIES

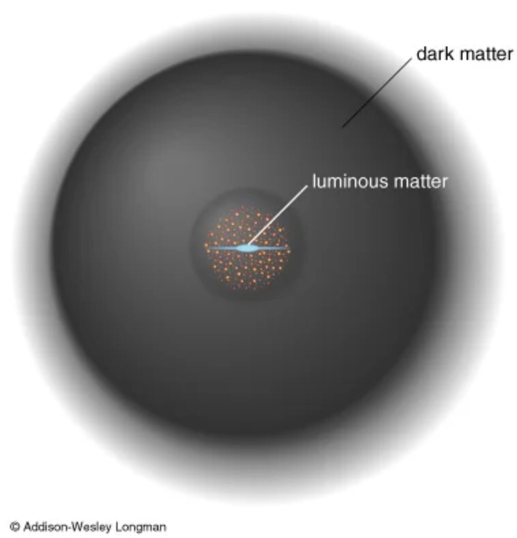

FIG. 1. Galaxies are surrounded by large and massive dark matter halos.

It is believed that all galaxies are surrounded by a large *dark matter halo*, essentially large balls of dark matter which contain luminous galaxies near their centers (see Fig. 1). As shown in this figure, dark matter halos extend way beyond the edge of their luminous central galaxy. Taken as a whole, the mass of the dark matter halo is quite a bit larger than the mass of the visible matter (stars + gas) within the galaxy.

The presence of an apparent large amount of dark matter within galaxies was discovered by Rubin & Ford, and their discovery was instrumental in solidifying the fact that dark matter is prevalent throughout the cosmos. Essentially, they were measuring the rotation curves of galaxies, looking at how fast stars rotate around the center of their respective galaxy. In Newtonian mechanics, such rotation curves, at a distance r from the center of the galaxy, should be given by

$$v(r) = \sqrt{\frac{GM(<r)}{r}}, \tag{1}$$

where $M(<r)$ is the mass contained within the radius r. Rubin and collaborators were able to measure the rotation speed of stars at the very edge of the galactic disk of several galaxies. If baryons were the only matter in the Universe, we would expect $M(<r)$ to asymptote to a constant there since there is no visible matter beyond the edge of the galactic disk. This would result in rotation curves falling of as $v \propto 1/\sqrt{r}$ at the edge of a disk galaxy. Instead, they measured approximately *flat* rotation curves there, $v \sim$ constant. This implies that we must have $M(<r) \propto r$ at the edge of galaxies. What can contribute to this rising contained mass as a function of radius? Dark matter. Figure 2 shows how dark matter can make the rotation curve much flatter than the naive prediction from just counting the mass of the stars and gas.

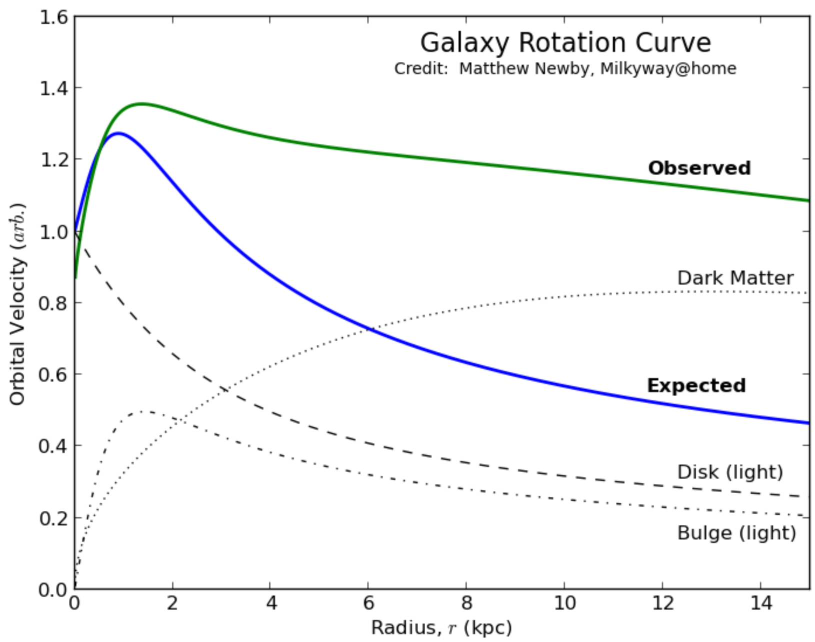

FIG. 2. Rotation curve of a galaxy

A. The Navarro-Frenk-White density profile

The fact that the galaxy rotation curves are flat at large radii puts constraints on the dark matter density profile. If we want $v \sim$ constant on the outskirt of disk galaxies, we must have $M(<r) \propto r$. Since the mass enclosed is given by

$$M(<R) = 4\pi \int_0^R r^2 dr \rho_{\rm DM}(r), \tag{2}$$

this means that we must have $\rho_{\rm DM}(r) \propto 1/r^2$ near the edge of disk galaxies. In the mid 1990s, Navarro, Frenk, and White simulated the structure of cold dark matter halos. They found an universal profile shape that seems to fit halos of all mass scales

$$\rho_{\rm NFW}(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}, \tag{3}$$

where r_s is called the *scale radius*, and ρ_s is a normalization parameter. At small radii $r \ll r_s$, this profile goes as $\rho(r) \propto 1/r$, while at large radii $r \gg r_s$, the profile goes as $\rho(r) \propto 1/r^3$. This is interesting since the only part of the profile that scales approximately as $1/r^2$ occurs near $r \sim r_s$. In fact, the slope of the profile is exactly -2 when $r = r_s$. This can be seen by computing the slope of the profile

$$\frac{d \ln \rho_{\rm NFW}}{d \ln r} = -1 - \frac{2(r/r_s)}{1+(r/r_s)}, \tag{4}$$

which is equal to -2 when $r = r_s$. This means that the edge of disk galaxies must always lie near $r = r_s$, which might seem like a fine-tuning. However, galaxy formation simulations indicate that this is indeed the case.