ASTR 425/525 Cosmology
Energy Content of the Universe

(Dated: September 24, 2025)

I. ENERGY CONTENT AND THE FRIEDMANN EQUATION

The Friedmann equation always relates the energy content of the Universe to the Hubble rate of expansion

$$H^2(z) = H_0^2 \left[\Omega_{\rm rad}(1+z)^4 + \Omega_{\rm m}(1+z)^3 + \Omega_K(1+z)^2 + \Omega_\Lambda\right], \tag{1}$$

where we have assumed a universe populated by radiation ($\Omega_{\rm rad}$), matter ($\Omega_{\rm m}$), and dark energy (Ω_Λ), while allowing for the presence of spatial curvature (Ω_K). Remember that we have the constraint

$$\Omega_K = 1 - \sum_i \Omega_i, \tag{2}$$

where the sum includes everything except for the curvature term. It is now time to elaborate a bit more on the physical components making up these various energy density parameters.

II. RADIATION

We define radiation as anything that has an equation of state $w = 1/3$, or more explicitly $p = \rho/3$. In our Universe, the principal contributors to the radiation energy density are photons and relativistic neutrinos, with possible minor contributions from yet undiscovered relativistic particles beyond the Standard Model of particle physics. Let's consider these in turns.

A. Photons

Photons are the largest contributor to the radiation density for the vast majority of the history of the Universe. While stars and other stellar objects have emitted plenty of photons since the Big Bang, the photon budget of the Universe is *entirely dominated* by what we call today the Cosmic Microwave Background (CMB). These are photons that were released in the early Universe when the first atoms formed and the Universe finally became transparent to these photons (around $z \sim 1100$). These photons have been redshifting since that epoch, and now have a frequency spectrum peaking in the microwave range of the electromagnetic spectrum (hence the name CMB). These photons appear to have a perfect thermal (blackbody) spectrum with a temperature today of

$$T_0 = 2.725\,{\rm K} = 2.348 \times 10^{-4}\,{\rm eV}. \tag{3}$$

Today, photons have a density given by

$$\Omega_\gamma = \frac{\rho_\gamma(t_0)}{\rho_{\rm c}} = 2.47 \times 10^{-5} h^{-2}, \tag{4}$$

where h is the reduced Hubble constant $h = H_0/(100{\rm km/s/Mpc})$.

B. Relativistic neutrinos

The Standard Model of particle physics contains 3 different flavors of neutrinos (electron, muon, tau). While we know that at least two of these have a non-zero mass, their masses are small enough that they plays a relatively small (although important) role in cosmology. In the early Universe when the temperature was significantly higher than than their masses, neutrinos behave as radiation. If we were to neglect the small mass, the density parameters for massless neutrino would be

$$\Omega_\nu = \frac{\rho_\nu(t_0)}{\rho_{\rm c}} = 1.68 \times 10^{-5} h^{-2} \left(\frac{N_{\rm eff}}{3}\right), \tag{5}$$

where $N_{\rm eff}$ is a factor introduced to take into account the details of how neutrinos decouple from the cosmic plasma. This number is not exactly equal to 3, but is instead $N_{\rm eff} = 3.044$ in the Standard Model (more on that later). While this expression is not accurate at late cosmological times (due to the fact that neutrinos do have masses), it is sufficient when performing early-Universe calculation since neutrinos have a temperature at that epoch that is way larger than their masses.

C. Light relics beyond the Standard Model

Many scenarios for physics beyond the Standard Model predict the existence of new massless (or nearly massless) particles. These behave as radiation and should be added to $\Omega_{\rm rad}$. The usual convention to take these into account is to absorb them into the $N_{\rm eff}$ value appearing in Eq. (5). Thus, when considering $N_{\rm eff} > 3.044$, we are generally referring to the presence of additional light relics beyond the regular photons and neutrinos.

III. MATTER

We define matter as anything that is pressureless, that is, has an equation of state $w = 0$. In our Universe, we have three distinct contributors to the matter density: baryons dark matter, and (at late times) massive neutrinos. Let's consider these in turns.

A. Baryons

In cosmology, baryons refer to the "normal" matter we are familiar with: the atoms and molecules making up our bodies, our planet, our Sun, and all other stars and planets populating the cosmos. Technically, baryons are bound states of three quarks (like protons and neutrons), but in cosmology we refer to all visible matter (including electrons which are technically *leptons*) as "baryons". The main reason for that protons and neutrons are about 2000 times more massive than electrons, so most of the mass of the visible matter in the Universe is indeed from baryons. Most of the baryonic mass in the Universe is in the form of hydrogen gas ($\sim 75\%$) and helium gas ($\sim 25\%$), with trace amounts of other elements with higher atomic numbers.

The surprising element though is that baryons form a small fraction of the overall matter content of the Universe. The latest CMB measurements from the Planck satellite yield

$$\Omega_{\rm b} h^2 = 0.02237, \tag{6}$$

which, using $h = 0.6736$ (also from Planck), gives $\Omega_{\rm b} = 0.0493$. So baryons form no more than $\sim 5\%$ of the critical density of the Universe today. However, independent measurements of the matter density parameter $\Omega_{\rm m}$ usually yield values ~ 0.3, showing that baryons are not only a small fraction of the critical density of the Universe today, but also a small fraction of the overall *matter* content of the Universe. So, there must be another dominant matter component in the Universe: dark matter.

B. Dark Matter

Nobody knows what dark matter is. All we know is that it behaves like matter (i.e. it has $w = 0$ and dilutes as a^{-3} with the expansion of the Universe). We often refer to dark matter as *cold dark matter* (CDM) to emphasize the fact that it has no pressure. Dark matter could be made of a yet unknown particle (or particles), or be made of small black holes that formed early in the history of the Universe. The Planck satellite measures the abundance of dark matter to be

$$\Omega_{\rm c} h^2 = 0.1200, \tag{7}$$

which, using $h = 0.6736$ (also from Planck), gives $\Omega_{\rm c} = 0.2644 \sim 5\Omega_{\rm b}$. So, dark matter is about 5 times as abundant as regular visible matter. Dark matter plays a fundamentally important role in the formation of structure in our Universe, as we will discuss later in this course.

C. Massive neutrinos

At late times, the neutrino temperature falls below the mass of at least two of the neutrino species, turning them into nonrelativistic particles with $w \approx 0$, hence contributing to the matter density. Neutrino oscillation experiments put a lower limit on the sum of neutrino masses

$$\sum m_\nu \geq 0.058\,\mathrm{eV}, \tag{8}$$

where we have assumed the normal mass ordering. It turns out that the density parameter for massive neutrinos is given by

$$\Omega_\nu h^2 = \frac{\sum m_\nu}{94\,\mathrm{eV}}, \tag{9}$$

meaning that massive neutrinos contribute a minimum of $\Omega_\nu h^2 > 6.2 \times 10^{-4}$ to the density today. Independently, cosmological data put an upper on the sum of neutrino masses of about

$$\sum m_\nu \lesssim 0.1\,\mathrm{eV}, \tag{10}$$

which means that we must also have $\Omega_\nu h^2 \lesssim 1.1 \times 10^{-3}$. Massive neutrinos thus form a tiny fraction of the matter density today (and they cannot be the dark matter!).

IV. DARK ENERGY

We refer to "dark energy" to denote a component of the Universe with $w \approx -1$ (leading to a nearly constant energy density that doesn't dilute very much with the expansion). If ρ_Λ is truly constant with time, we often refer to dark energy as the *cosmological constant* (CC) Λ, in homage to Einstein who introduced such constant (with the inverse sign) to make the Universe static instead of expanding. This Λ term was simply added to the right-hand side of the Friedmann equation as follows

$$H^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}. \tag{11}$$

But this seems a little arbitrary and we instead define an associated energy density

$$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G}, \tag{12}$$

and associated density parameters

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_\mathrm{c}} = \frac{\Lambda}{3H_0^2}. \tag{13}$$

The Planck satellite yields $\Omega_\Lambda = 0.6847$, which results in $\rho_\Lambda = 2.5 \times 10^{-11}$ eV^4 (in natural units where $\hbar = c = 1$). But even this CC needs an explanation for its physical origins. The simplest explanation is that it is *vacuum energy*, that is, the intrinsic zero-point energy of empty space. But this interpretation immediately runs into problems as the natural value for this vacuum energy is $\rho_\Lambda \sim M_\mathrm{pl}^4 \sim 10^{112}$ eV^4, where M_pl is the Planck mass. This is the famous ~ 120 orders of magnitude discrepancy between the observed and theoretically motivated value of the dark energy density. This is one of the most intractable problems in physics.

Another possibility is that dark energy is dynamical, but changes on such long time scales that it appears constant to us. A scalar field that is slowly relaxing to the minimum of some potential would do the trick, but model-building such model in a self-consistent way remains a challenge.