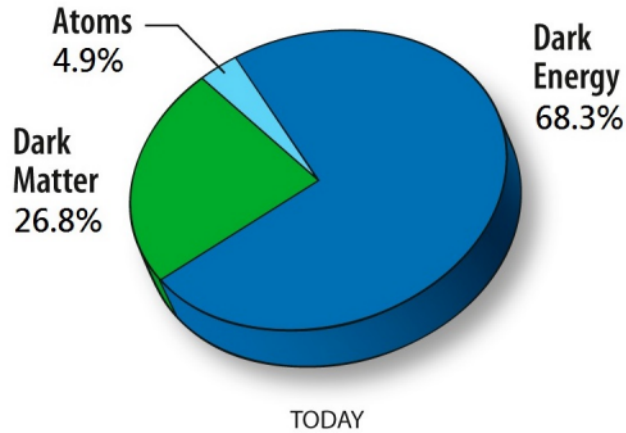


Question 1

Today, the fractional contribution of the main energy components populating the Universe roughly follows the pie chart below:



where the contribution from photons and massless neutrinos are too small to be shown here. Here, “atoms” refers to baryons (normal visible matter). This pie chart was very different in the past. Draw what the cosmological pie chart looked like at $z = 1100$, where the cosmic microwave background photons were emitted. Show the fractional contributions from photons, massless neutrinos, dark matter, and baryons. Is dark energy relevant at this redshift?

From the pie chart we observe that

$$\Omega_{b,0} = 0.049 \quad (1)$$

$$\Omega_{\text{cdm},0} = 0.269 \quad (2)$$

$$\Omega_{\Lambda,0} = 0.683 \quad (3)$$

Where $\cdot_0$ stands for “today.” Taking $H_0 \simeq 67 \text{ km/s/Mpc}$ (i.e., $h = 0.67$), we can compute the density parameters for the radiation components. For massless neutrinos:

$$\begin{aligned} \Omega_{\nu,0} &= 1.68 \times 10^{-5} h^{-2} \left(\frac{N_{\text{eff}}}{3} \right) \\ (N_{\text{eff}} = 3.044) \quad &= 3.797 \times 10^{-5} \end{aligned} \quad (4)$$

And for photons:

$$\begin{aligned} \Omega_{\gamma,0} &= 2.47 \times 10^{-5} h^{-2} \\ &= 5.502 \times 10^{-5} \end{aligned} \quad (5)$$

Now that we have the values today, we can use their scaling laws:

$$\begin{aligned}
\Omega_{\Lambda}(z) &= \left(\frac{H_0}{H(z)} \right)^2 \Omega_{\Lambda,0} \\
\Omega_b(z) &= \left(\frac{H_0}{H(z)} \right)^2 (z+1)^3 \Omega_{b,0} \\
\Omega_{\text{cdm}}(z) &= \left(\frac{H_0}{H(z)} \right)^2 (z+1)^3 \Omega_{\text{cdm},0} \\
\Omega_{\nu}(z) &= \left(\frac{H_0}{H(z)} \right)^2 (z+1)^4 \Omega_{\nu,0} \\
\Omega_{\gamma}(z) &= \left(\frac{H_0}{H(z)} \right)^2 (z+1)^4 \Omega_{\gamma,0}
\end{aligned} \tag{6}$$

Using

$$H(z) = H_0 \sqrt{\Omega_{\Lambda} + \Omega_{b,0}(z+1)^3 + \Omega_{\text{cdm},0}(z+1)^3 + \Omega_{\gamma,0}(z+1)^4 + \Omega_{\nu,0}(z+1)^4} \tag{7}$$

We can compute

$$\Omega_{\Lambda}(z = 1100) = 1.21 \times 10^{-9} \tag{8}$$

$$\Omega_b(z = 1100) = 0.116 \tag{9}$$

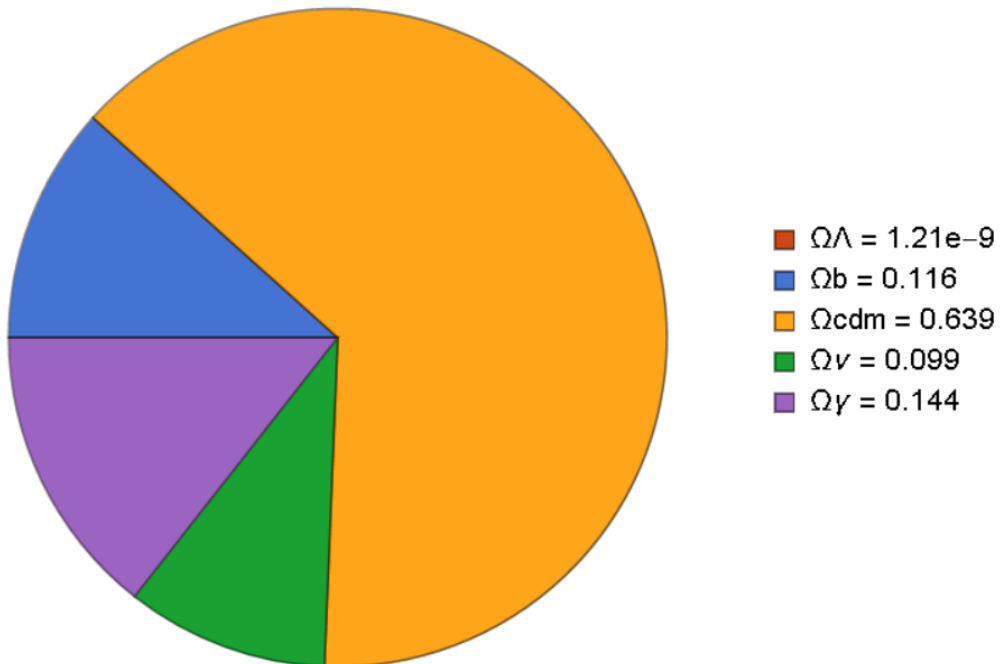
$$\Omega_{\text{cdm}}(z = 1100) = 0.639 \tag{10}$$

$$\Omega_{\nu}(z = 1100) = 0.0994 \tag{11}$$

$$\Omega_{\gamma}(z = 1100) = 0.1441 \tag{12}$$

Which produces the following pie chart:

Density parameters at z = 1100



Over time, the density parameters evolve (up until $z = 1100$)

