Question 1

Big Bang Nucleosynthesis (BBN), together with the measured abundance of ^{4}He, is an excellent probe of whether the physics of the early Universe was the same as today. Consider the following changes to the physics of the early Universe and determine whether they would result in an increased or decreased ^{4}He abundance in the early Universe.

a) A larger neutron lifetime τ_n.

b) A larger value of $Q \equiv m_\mathrm{n} - m_\mathrm{p}$, the mass difference between a neutron and a proton.

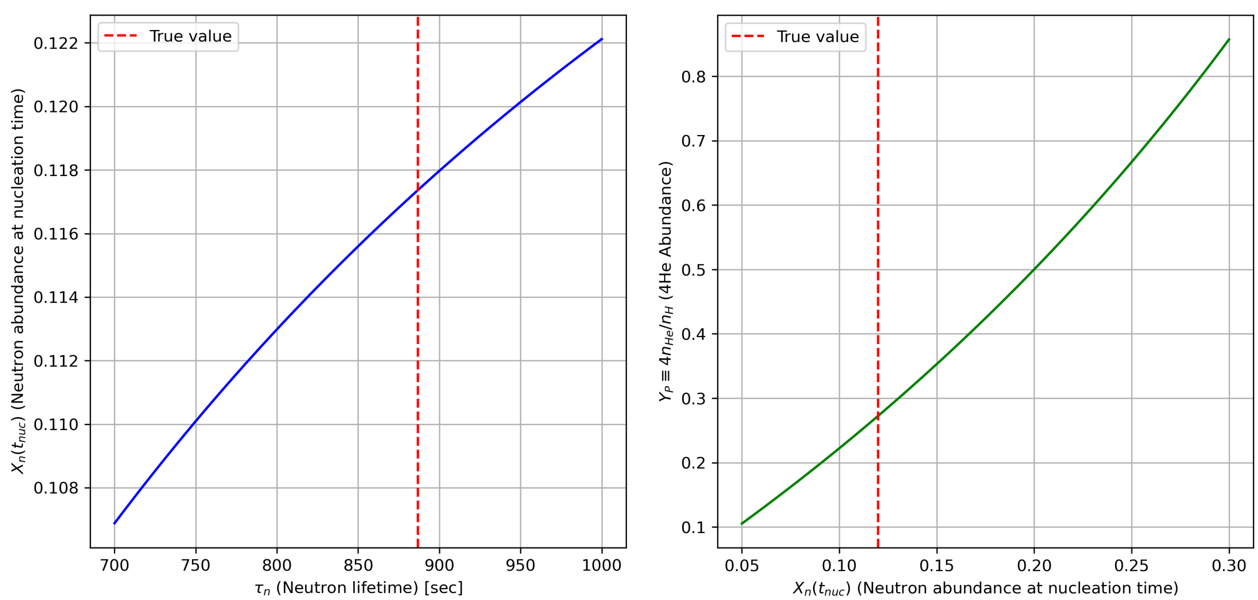
c) A larger value of $g_*(T)$ at all times.

d) An increased value of the baryon-to-photon ratio, η_b.

a) Recall that other physical processes (which will be relevant in later parts of this worksheet) tell us that t_nuc, the time we have to wait to start forming composite nuclei in our universe (also known as the *deuterium bottleneck*) is ~ 5 minutes and independent of the neutron lifetime τ_n.

The value of t_nuc is used along with τ_n to compute the abundance of neutrons (at this time). With the real value of τ_n, we had $X_n(t_\mathrm{nuc}) \simeq \frac{1}{6} e^{-t_\mathrm{nuc}/\tau_\mathrm{n}} \approx 0.12$. If we increase the neutron lifetime, then the abundance of neutrons at t_nuc increases. This makes sense, as there will be less decays in this window of time, hence a larger abundance.

With a larger abundance X_n, we get a larger ratio of $n_\mathrm{He}/n_\mathrm{H}$ and thus a larger ^{4}He abundance.



b) Recall that Q was introduced when talking about the ratio between neutrons and protons:

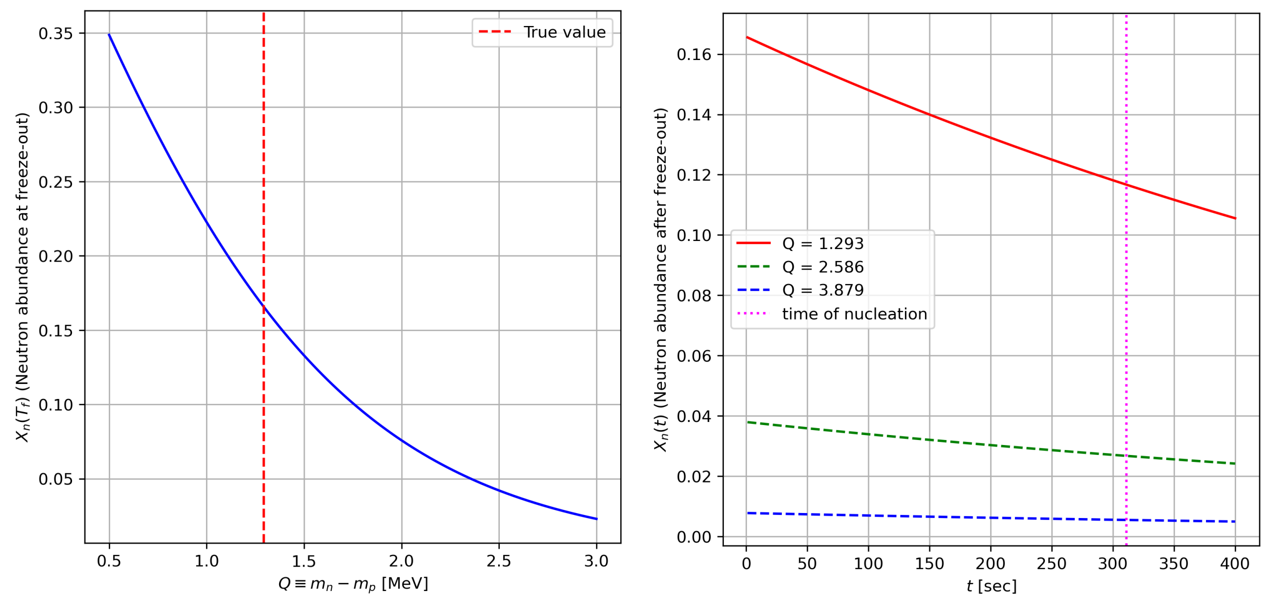
$$\left(\frac{n_\mathrm{n}}{n_\mathrm{p}}\right)_\mathrm{eq.} = e^{-Q/T} \tag{1}$$

Further, we showed that the neutron fraction $X_\mathrm{n} = \frac{n_\mathrm{n}}{n_\mathrm{n}+n_\mathrm{p}}$ depends on Q as follows:

$$X_{\mathrm{n,eq}}(T) = \frac{e^{-Q/T}}{1+e^{-Q/T}} \tag{2}$$

Relevant to us is the evaluation of this abundance at $T = T_\mathrm{f}$, the temperature when the weak interaction ate and the Hubble expansion were equal to each other. At fixed T_f, we see that the neutron abundance at freeze-out decreases when Q increases.

A lower abundance at freeze-out implies a lower *initial value* for $X_\mathrm{n}(t) = X_{\mathrm{n,eq}}(T_\mathrm{f})e^{-t/\tau_\mathrm{n}}$, so at $t = t_\mathrm{nuc}$, X_n is smaller, and as we saw in part (a) this implies a smaller abundance.

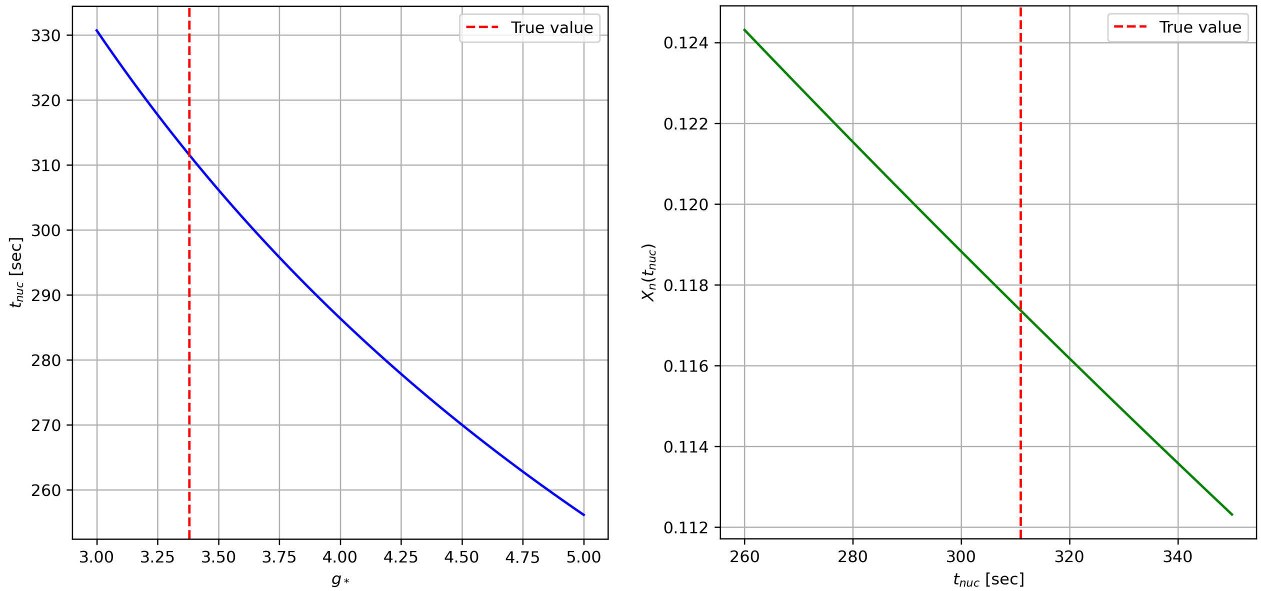


c) After finding the temperature required for nucleosynthesis, T_nuc, we computed that the age of the universe (at this temperature) is given by

$$\frac{t}{\mathrm{sec}} \simeq \frac{2.42}{\sqrt{g_*}}\left(\frac{T}{\mathrm{MeV}}\right)^{-2} \tag{3}$$

At face value, a smaller g_* gives a larger t_nuc and a larger g_* gives a smaller t_nuc. A smaller t_nuc gives a larger X_n, so a larger abundance.

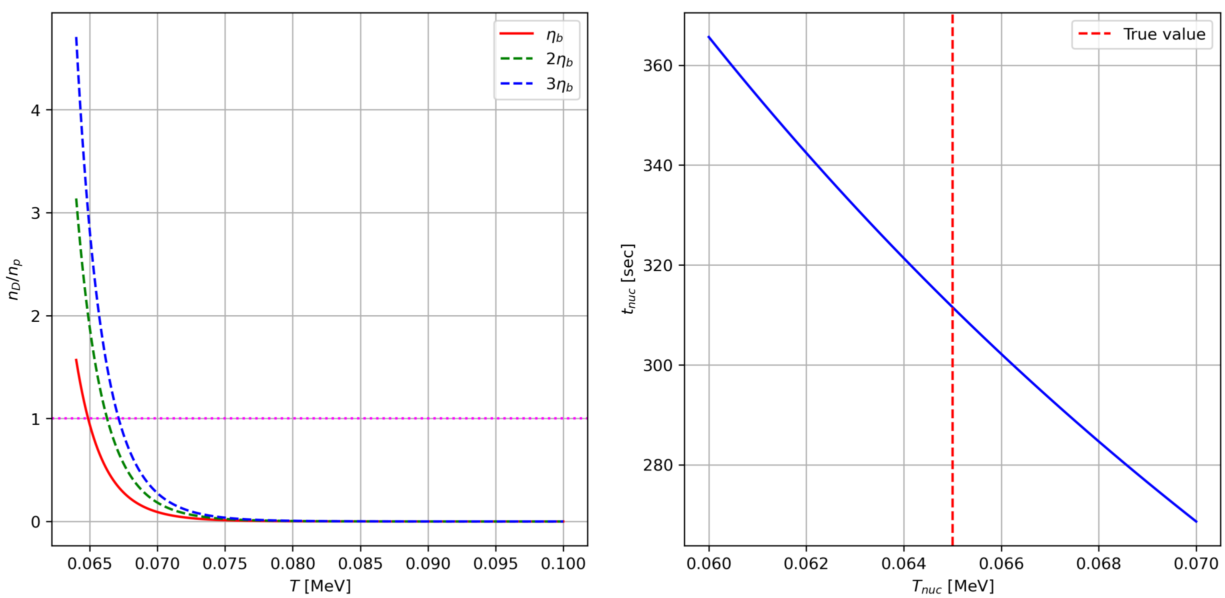
This makes physical sense: If t_nuc is smaller, this gives neutrons a smaller time window to decay, so their (neutron) abundance is larger, hence a larger abundance of ^{4}He. Further, the smaller t_nuc is a direct consequence of the evolution of the universe: a larger $g_*(T)$ at all times implies a larger radiation energy density, which in turn yields a larger Hubble rate, so it takes less time for the temperature to drop to T_nuc (larger g_* gives younger universes at fixed temperature).



d) Recall that η_b dictates the temperature needed to start nucleosynthesis:

$$\left(\frac{n_\mathrm{D}}{n_\mathrm{p}}\right)_\mathrm{eq.} \approx 4\eta_\mathrm{b}\left(\frac{T}{m_\mathrm{p}}\right)^{3/2} e^{B_\mathrm{D}/T} \tag{4}$$

Where T_nuc is the temperature at which the deuterium abundance grows large enough: $n_\mathrm{D}/n_\mathrm{p} \sim 1$. If we increase η_b, then the required T_nuc increases as well.



A higher T_nuc gives a smaller time for nucleosynthesis t_nuc. As such, X_n is larger, as we are giving neutrons a smaller time window to decay. We conclude that a larger η_b produces a larger abundance of ^{4}He.