Problem 1

Homogeneity and isotropy refer to very different aspects of the Universe. Here, we want to explore how having one of these properties does not necessarily implies the other

a) Sketch a universe that is homogeneous but not isotropic. Can you think of a physical situation where this could arise?

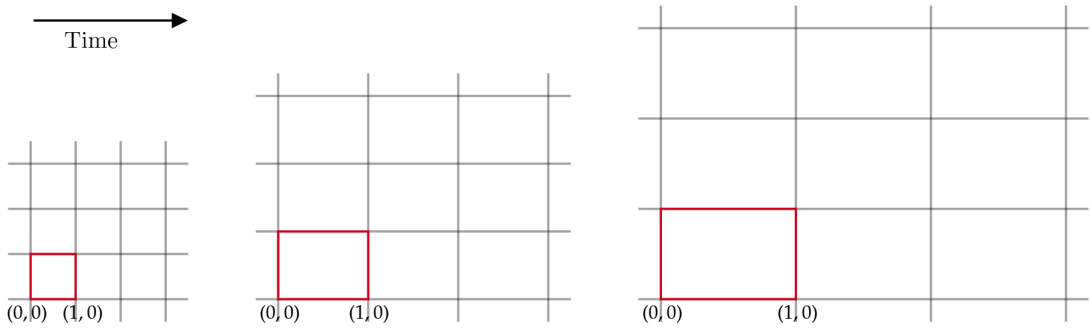
b) Sketch a universe that is isotropic but not homogeneous. Can you think of a physical situation where this could arise?

Homogeneous universes are those that look the same at every point in space (i.e., there is no preferred location). Isotropic universes are those that look the same in every direction (i.e., there is no preferred direction in the sky).

a) This scenario occurs in a universe in which we have different expansion rates in each direction:

$$ds^2 = -dt^2 + a_x^2(t)dx^2 + a_y^2(t)dy^2 + a_z^2(t)dz^2 \tag{1}$$

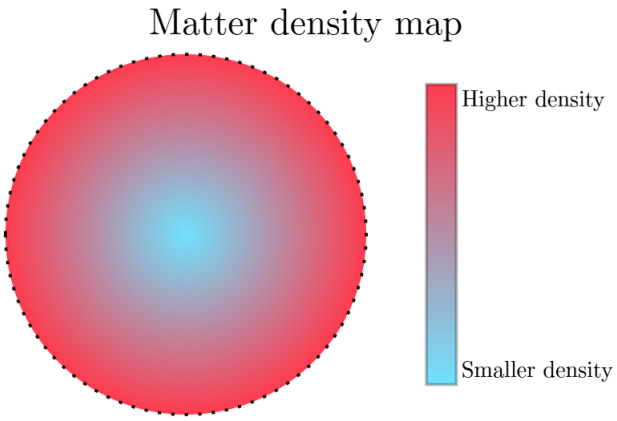
In a universe that is uniformly filled with matter. This universe isn't isotropic, because directions can be distinguished by their different rates of expansion.



Fun fact: The study of Bianchi cosmologies is the study of homogeneous (not necessarily isotropic) cosmologies. Universes in which the expansion rate is different in each direction are known as Kasner-like universes.

b) Consider a universe in which the matter distribution is radially symmetric around a point. That is, the density depends on the distance from that point but looks the same in every direction.

For example:



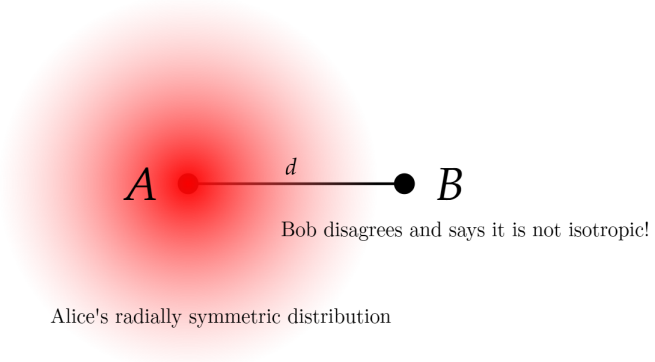
From the central point, the universe is (looks) isotropic, but it is not homogeneous since there is more matter the further away we look.

Problem 2

In this question, we explore how homogeneity and isotropy can be related under certain circumstances. Argue that if a universe appears isotropic to two distinct observers separated by some distance d, then that universe must be homogeneous.

Call the observers Alice and Bob. From our discussion of part (b) in problem 1, we know that Alice and Bob observe radially-symmetric distributions around them.

If the distribution of Alice is not uniform, then it would not look isotropic from Bob's point of reference in space.



Thus, the only way for both of them to say that they are in an isotropic universe is for their universe to be homogeneous.