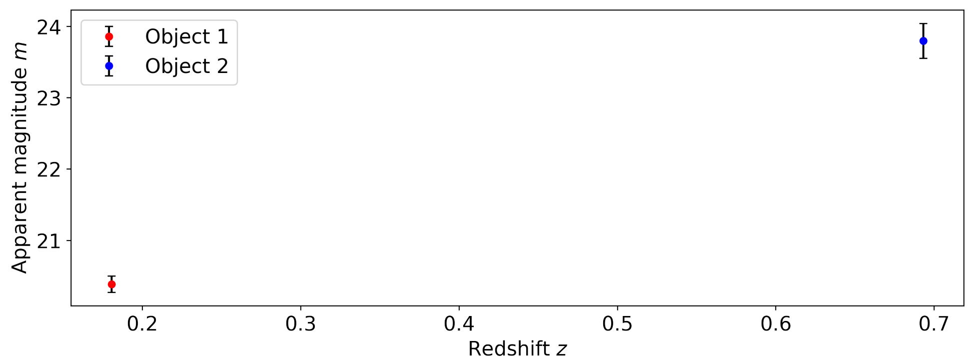
Question 1

Even without knowing the absolute luminosity of certain astronomical objects, it is possible to extract important cosmological information from the *difference* in apparent magnitude between two similar objects. Consider two Type Ia supernovae, one at $z_1 = 0.1806$ and apparent magnitude $m_1 = 20.3864 \pm 0.1142$, and the other at $z_2 = 0.69315$ with apparent magnitude $m_2 = 23.7964 \pm 0.244$.

a) Deriva an expression for the ration $d_L(z_2)/d_L(z_1)$ as a function of $m_2 - m_1$. Argue that ratio is independent of H_0.

b) Use the expression derived in part (a) to show that a cosmological model with $\Omega_\mathrm{m} = 0.3$ and $\Omega_\Lambda = 0.7$ is a much better fit to the data than a flat matter-dominated universe with $\Omega_\mathrm{m} = 1$. This argument was instrumental to the discovery of dark energy.



a) Recall that (where $\mu = m - M$)

$$\mu = 5\log_{10}\left(\frac{d_L}{\mathrm{Mpc}}\right) + 25 \tag{1}$$

We can exponentiate (base 10) both sides to see that

$$10^{\mu} = 10^{25}\left(\frac{d_L}{\mathrm{Mpc}}\right)^5 \tag{2}$$

Consider equation 2 for both data points. Dividing one from the other results in

$$\begin{aligned}
\frac{10^{\mu_2}}{10^{\mu_1}} &= \frac{10^{25}\left(\frac{d_L(z_2)}{\mathrm{Mpc}}\right)^5}{10^{25}\left(\frac{d_L(z_1)}{\mathrm{Mpc}}\right)^5} \\
10^{\mu_2-\mu_1} &= \left(\frac{d_L(z_2)}{d_L(z_1)}\right)^5 \\
\boxed{M_1 \simeq M_2} \qquad 10^{m_2-m_1} &= \left(\frac{d_L(z_2)}{d_L(z_1)}\right)^5 \\
10^{(m_2-m_1)/5} &= \frac{d_L(z_2)}{d_L(z_1)}
\end{aligned} \tag{3}$$

Although $d_L(z)$ has H_0-dependence, the ratio d_L/d_L eliminates this dependence explicitly.

b) Using Eq. 3, we can use the data to compute the left hand side:

$$10^{(m_2-m_1)/5} = 4.808 \tag{4}$$

The right hand side depends on the choice of cosmological model. Recall that

$$\begin{aligned} d_L(z) &= (z+1)S_k(z) \\ \text{(In a spatially-flat universe)} \qquad &= (z+1)\chi(z) \\ &= (z+1)\int_0^z \frac{dz'}{H(z')} \end{aligned} \tag{5}$$

In the universe with no dark energy

- $H(z) = H_0\sqrt{(z+1)^3} = H_0(z+1)^{3/2}$.
- The luminosity distances follow:

$$\begin{aligned} d_L(z_2) &= 0.783\ H_0^{-1} \\ d_L(z_1) &= 0.188\ H_0^{-1} \end{aligned}$$

- Ratio of luminosity distnaces

$$\frac{d_L(z_2)}{d_L(z_1)} = 4.1675 \tag{6}$$

In the universe with dark energy

- $H(z) = H_0\sqrt{\Omega_\mathrm{m}(z+1)^3 + \Omega_\Lambda}$.
- The luminosity distances follow:

$$\begin{aligned} d_L(z_2) &= 0.982\ H_0^{-1} \\ d_L(z_1) &= 0.204\ H_0^{-1} \end{aligned}$$

- Ratio of luminosity distnaces

$$\frac{d_L(z_2)}{d_L(z_1)} = 4.81 \tag{7}$$

We see that the ratio of luminosity distances from a universe with dark energy matches better with the data prediction shown in Eq. 4.