

ASTR 425/525 Cosmology

Worksheet #7
Wednesday 09/10/2025

Question 1.

In the following, express the different ages as functions of the Hubble time, $1/H_0$.

- (a) Compute the age of a flat ($k = 0$) universe entirely dominated by matter ($\Omega_m = 1$, all other density parameters are zero).
- (b) Compute the age of a flat ($k = 0$) Universe with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$. You can do the integral numerically if you want, although it also has an analytical solution (see below). Is this Universe younger or older than the purely matter-dominated universe of part (a)?
- (c) Compute the age of a curved matter-dominated universe with $\Omega_m = 0.9$ and $\Omega_K = 0.1$? You can Taylor expand the integrand to perform the integral. Is this universe younger or older than a purely matter-dominated flat ($k = 0$) universe?

Hint: you may find this integral useful:

$$\int_0^1 \frac{\sqrt{x}}{\sqrt{1+bx^3}} dx = \frac{2}{3\sqrt{b}} \sinh^{-1}(\sqrt{b}) \quad (1)$$

