Question 1

In a famous 1968 paper (which he basically got the Nobel prize for), Peebles argues that recombination of an electron directly to the ground state of an hydrogen atom would not lead to a net decrease of the ionization fraction X_e. Instead, recombination has to proceed through the excited states, especially the $2s$ and $2p$ states of Hydrogen.

a) Could you think of a reason why recombination directly to the ground state (accompanied by the emission of a 13.6 eV photon) would not lead to a direct decrease of X_e? Think about what happens to that 13.6 eV photon.

b) If an electron recombines into the $2s$ state, how does it get to the ground state ($1s$)? Is this an allowed transition? Think about selection rules and angular momentum conservation. How many photons are emitted in this $2s \to 1s$ transition?

c) If an electron recombines into the $2p$ state, how does it get to the ground state ($1s$)? What kind of photons are emitted in this transition? Does this $2p \to 1s$ transition lead to a net recombination event? Under which conditions?

a) If recombination into the ground state emits a 13.6 eV photon ($\gamma_{13.6}$), then there would be no net recombination, as the emitted photon $\gamma_{13.6}$ won't have to travel much to ionize another hydrogen atom.

$$e^- + p \to H + \gamma_{13.6} \quad \Leftrightarrow \quad \gamma_{13.6} + H \to e^- + p$$

Where it is relevant to note that these photons had an incredibly large cross section (equivalently, a very short mean free path).

b) Recall that 1-photon transitions are allowed if

$$\Delta\ell = \pm 1$$

Where ℓ denotes the angular momentum of a given state. Since $2s$ and $1s$ are both states with $\ell = 0$, $\Delta\ell = 0$ hence it is forbidden. But this is precisely the key: we can use two photons to carry away a net zero angular momentum:

$$e^- + p \to H + 2\gamma$$

These photons will split the energy, yielding less energetic photons that can't be reabsorbed by other hydrogen atoms. This way, we get a net recombination process.

c) The p in $2p$ tells us that $\ell = 1$ (in the same way that s in $1s$ tells us that $\ell = 0$). As such, the $2p \to 1s$ transition is allowed by single-photon emission.

The emitted photons have the energy to excite the $1s$ (ground state) Hydrogen atoms again into $2p$. This delays the creation of a net amount of ground state Hydrogen, which is what we seek to get from recombination.

Nonetheless, the expansion of the universe will reduce the energy of these photons through redshifting, effectively putting them out of the $1s \to 2p$ resonance and thus obtaining a stable number of ground state hydrogen atoms.