PHYS 301
Thermodynamics and Statistical Mechanics

Homework Assignment 10

Due date: Monday April 27 2026 5pm, submitted on UNM Canvas

Question 1 (3 points).

Enthalpy: To create a system with total energy E on Earth, we not only need to provide the energy E but also the work needed to make room for it (e.g. displacing the air that used to be where the system now sits). At constant pressure P, this work is simply PV, where V is the volume of the newly created system. To take this into account, we define the *enthalpy* as $H = E + PV$.

(a) Using the first law of thermodynamics with $đQ = TdS$, show that $dH = TdS + VdP$.

(b) The heat capacity at *constant pressure* C_P is defined as

$$C_P = \frac{đQ}{dT}\Big|_P. \tag{1}$$

Argue that this heat capacity is related to the enthalpy via

$$C_P = \frac{\partial H}{\partial T}\Big|_P. \tag{2}$$

Question 2 (12 points).

Joule-Thomson process for cooling gases: Figure 1 below shows a thermally insulated pipe (i.e. no heat flow) which has a porous barrier separating two halves of the pipe. A gas of volume V_1, initially on the left-hand side of the pipe, is forced by a piston to go through the porous barrier using a constant pressure P_1. Assume the process can be treated quasistatically. As a result the gas flows to the right-hand side, resisted by another piston which applies a constant pressure P_2 ($P_2 < P_1$). Eventually all of the gas occupies a volume V_2 on the right-hand side.

(a) Show that the enthalpy $H = E + PV$ is conserved in the Joule-Thomson process, where E is the total energy of the gas.

(b) Define the Joule-Thomson coefficient as

$$\mu_{\rm JT} \equiv \frac{\partial T}{\partial P}\Big|_H. \tag{3}$$

Use enthalpy conservation ($dH = 0$) and the results from question 1 to show that this coefficient takes the form

$$\mu_{\rm JT} = \frac{1}{C_P}\left[T\left(\frac{\partial V}{\partial T}\right)_P - V\right]. \tag{4}$$

You will need the following relation: $(\partial S/\partial P)_T = -(\partial V/\partial T)_P$.

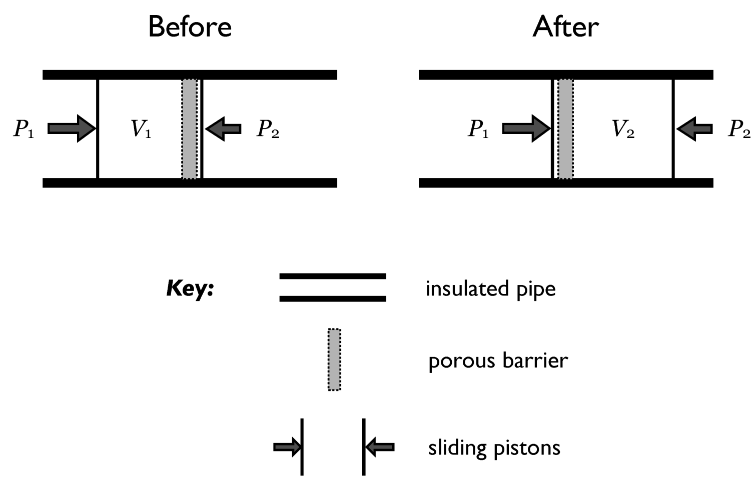

Figure 1: The Joule-Thomson process

(c) Compute $\mu_{\rm JT}$ for an ideal gas. Can the Joule-Thomson process be used to cool an ideal gas?

(d) If we wish to use the Joule-Thomson process to cool a real (non-ideal) gas, what must the sign of $\mu_{\rm JT}$ be?

(e) Derive $\mu_{\rm JT}$ for a gas obeying the van der Waals equation of state to leading order in the density N/V, where N is the number of gas particles. For what values of temperature T can the gas be cooled?

Question 3 (5 points).

In the course of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed *adiabatically* to a pressure of 7 atm. (Air is mostly diatomic nitrogen and oxygen.)

(a) What is the final volume of this air after compression?

(b) How much work is done in compressing the air?

(c) If the temperature of the air is initially 300 K, what is the temperature after compression?