PHYS 301
Thermodynamics and Statistical Mechanics

Homework Assignment 2

Due date: Sunday February 8 2026 5pm, submitted on UNM Canvas

Question 1 (3 points).

We saw in class that the multiplicity of an ideal gas with N particles, volume V, and total energy E is $\Omega(N, V, E) = \sigma(N, V)E^{3N/2}$, where $\sigma(N, V)$ is independent of E.

(a) Show that the relationship between total energy and temperature is

$$E = \frac{3}{2} N k_{\mathrm{B}} T. \tag{1}$$

(b) Compute the heat capacity of this system.

(c) Derive an expression for the change in entropy of this system as we heat the system from temperature T_1 to T_2.

Question 2 (8 points).

Consider an Einstein solid with N oscillators, each with natural frequency ω, sharing q quanta of energy. Neglecting the zero-point energy of the oscillators, the total energy of the system is

$$E_{\mathrm{tot}} = q\hbar\omega. \tag{2}$$

(a) Using the multiplicity of microstates for an Einstein solid, compute the entropy $S(N, q)$ of this system in the $N \gg 1$ limit, using Stirling's formula to simplify your answer.

(b) By writing the entropy as a function of N and E_{tot}, $S(N, E_{\mathrm{tot}})$, derive an expression for E_{tot} as a function of the temperature T.

(c) At which value of E_{tot} (and T) is the entropy maximized? Does this value make sense to you? Explain why.

(d) Explain physically the behavior of the system as $T \to 0$ and $T \to \infty$. Assuming $E_{\mathrm{tot}} > 0$, can an Einstein solid have a negative temperature?

Question 3 (4 points).

Consider a system of N **interacting** spins. At low temperatures, the interactions ensure that all spins are either aligned or anti-aligned with the z axis, even in the absence of an external magnetic

field. At high temperatures, the interactions become less important and spins can point in either $\pm z$ direction. If the heat capacity takes the form,

$$C = C_{\max}\left(\frac{2T}{T_0} - 1\right) \quad \text{for} \quad \frac{T_0}{2} < T < T_0 \quad \text{and} \quad C = 0 \quad \text{otherwise,} \tag{3}$$

determine $C_{\max}$.