

PHYS 301
Thermodynamics and Statistical Mechanics

Homework Assignment 3

Due date: Sunday February 15 2026 5pm, submitted on UNM Canvas

Question 1 (4 points).

Consider a system consisting of N independent spin-1/2 particles, each of which can be in one of two quantum states, ‘up’ and ‘down’. In a magnetic field B , the energy of a spin in the up state is $-\mu B/2$ and that of the down state is $+\mu B/2$, where μ is the magnetic moment of a particle.

(a) Show that the partition function is

$$Z = 2^N \cosh^N \left(\frac{\mu B}{2k_B T} \right) \quad (1)$$

(b) Find the average energy E and entropy S . Check that your results for both quantities make sense in at $T = 0$ and $T \rightarrow \infty$.

(c) Compute the magnetization M of the system as a function of temperature, defined by $M \equiv \mu \langle N_\uparrow - N_\downarrow \rangle$ where N_\uparrow, N_\downarrow are the number of up and down spins, respectively.

(d) The magnetic susceptibility is defined as $\chi \equiv \partial M / \partial B$. Derive Curie’s Law which states that at high temperatures $\chi(T) \propto 1/T$.

Question 2 (3 points).

A quantum violin string can vibrate at frequencies $\omega, 2\omega, 3\omega$ and so on. Each vibration mode can be treated as an independent quantum harmonic oscillator. Ignore the zero point energy, so that the mode with frequency $p\omega$ has energy $E = n\hbar p\omega$, $n \in \mathbf{Z}$. Was it the average energy of the string at temperature T ?

Question 3 (3 points).

Compute the entropy of a one-dimensional ideal gas with N particles of mass m at temperature T , confined to a line of length L .