PHYS 301
Thermodynamics and Statistical Mechanics

Homework Assignment 4

Due date: Sunday February 22 2026 5pm, submitted on UNM Canvas

Question 1 (4 points).

For a system of 1 particle that has nearly continuous microstates in 1 dimension, the partition function can be written as

$$Z_1 = \frac{1}{h} \int dx dp \, e^{-\beta H(x,p)}, \tag{1}$$

where $H(x,p)$ is the Hamiltonian of the system as a function of position x and momentum p.

(a) If the Hamiltonian is

$$H(x,p) = \frac{p^2}{2m} + \lambda x^4, \tag{2}$$

show that the heat capacity for a gas of N independent such particles is $C_V = 3Nk_B/4$.

(b) Explain why the heat capacity is the same regardless of whether the particles are distinguishable or indistinguishable.

Question 2 (3 points).

Cold interstellar molecular clouds often contain the molecule cyanogen (CN), whose first rotational excited states have an energy of 4.7×10^{-4} eV (above the ground state). There are actually three such excited states, all with the same energy. Supposed that a study of the absorption spectrum of starlight that passes through one these molecular clouds showed that for every ten CN molecules that are in the ground state, approximately three others are in the three first excited states (that is, an average of one in each of these states). Use this information to determine the temperature of the cloud. Here a more useful numerical value of the Boltzmann constant is $k_B = 8.62 \times 10^{-5}$ eV/K.

Question 3 (3 points).

For a system at fixed T (i.e. obeying the Boltzmann distribution), we discussed that the system has a well-defined average energy $\langle E \rangle$. The actual energy of the system can of course fluctuate about this average value. Show that these fluctuations in energy

$$\langle \Delta E^2 \rangle = \langle (E - \langle E \rangle)^2 \rangle = \langle E^2 \rangle - \langle E \rangle^2 \tag{3}$$

are always proportional to the heat capacity C_V. This means that the energy fluctuations in a system are related to its capacity to absorb (or dissipate) energy. This is known as the *fluctuation-dissipation theorem*.