PHYS 301
Thermodynamics and Statistical Mechanics

Problems #1

Wednesday, 01/21/2026

Question 1.

Suppose you flip 50 fair coins.

(a) How many possible microstates are there? How many macrostates?

(b) How many ways are there of getting exactly 25 heads and 25 tails?

(c) What is the probability of getting exactly 25 heads and 25 tails?

(d) What is the probability of getting exactly 30 heads and 20 tails?

(e) What is the probability of getting exactly 40 heads and 10 tails?

(f) What is the probability of getting exactly 50 heads?

(g) Sketch a graph of the probability of getting n heads, as a function of n.

Question 2.

An Einstein solid is a collection of N independent quantum harmonic oscillators with fundamental frequency ω. The ith harmonic oscillator in this system has energy

$$E_i = \left(n_i + \frac{1}{2}\right)\hbar\omega, \tag{1}$$

where n_i is an integer larger or equal to zero. For an Einstein solid, the macrostates can be labeled by the total energy of the system

$$E_{\text{tot}} = \sum_{i=1}^{N} E_i = \sum_{i=1}^{N}\left(n_i + \frac{1}{2}\right)\hbar\omega = \left(\sum_{i=1}^{N} n_i + \frac{N}{2}\right)\hbar\omega = \left(q + \frac{N}{2}\right)\hbar\omega, \tag{2}$$

where we have introduced the total number of energy quanta in the system as $q = \sum_{i=1}^{N} n_i$. Thus, macrostates can be labeled by q in the microcanonical ensemble.

(a) List all possible microstates for $N = 3$ oscillators sharing $q = 2$ energy quanta.

(b) List all possible microstates for $N = 3$ oscillators sharing $q = 3$ energy quanta.

(c) List all possible microstates for $N = 3$ oscillators sharing $q = 4$ energy quanta.

(d) Argue that for N oscillators sharing q energy quanta, the multiplicity of that macrostate is

$$\Omega(N, q) = \frac{(q + N - 1)!}{q!(N - 1)!}. \tag{3}$$