PHYS 301
Thermodynamics and Statistical Mechanics

Problems #11

Wednesday, 04/08/2026

Question 1.

Consider a gas of non-interacting ultra-relativistic electrons at some temperature T, chemical potential μ, and in volume V, whose mass may be neglected. Remember that electrons have spin degeneracy $g_s = 2$.

(a) Find an integral for the grand canonical potential $\Phi = -k_\mathrm{B} T \ln \mathcal{Z}$.

(b) Show that $PV = \langle E \rangle / 3$.

(c) Show that at zero temperature $PV^{4/3} = \text{const}$.