PHYS 301: Thermodynamics and Statistical Mechanics
Solutions to Problem Set #13

Question 1: Expanding Bubbles in a Lake

Problem Statement: Two identical bubbles rise to the surface of a lake. Bubble A rises quickly (no heat exchange), while Bubble B rises slowly (remains in thermal equilibrium with the water). Which bubble is larger by the time they reach the surface?

Solution: Let the bottom of the lake be the initial state i and the surface be the final state f. Both bubbles start with the same initial volume V_i, initial pressure P_i, and initial temperature T_i. At the surface, both bubbles experience the same final pressure P_f, where $P_f < P_i$.

We can model the gas inside the bubbles as an ideal gas.

Bubble B (Slow rise): Because it rises slowly, it remains in thermal equilibrium with the surrounding water. Assuming the water temperature is uniform, the expansion is **isothermal**. According to Boyle's Law ($PV = \text{const}$ for constant T):

$$P_i V_i = P_f V_B \implies V_B = V_i \left(\frac{P_i}{P_f} \right) \tag{1}$$

Bubble A (Fast rise): Because it rises quickly with no heat exchange ($\delta Q = 0$), the expansion is **adiabatic**. For an adiabatic expansion of an ideal gas, $PV^\gamma = \text{const}$, where $\gamma = C_P/C_V > 1$.

$$P_i V_i^\gamma = P_f V_A^\gamma \implies V_A^\gamma = V_i^\gamma \left(\frac{P_i}{P_f} \right) \implies V_A = V_i \left(\frac{P_i}{P_f} \right)^{1/\gamma} \tag{2}$$

Comparison: We are comparing V_B and V_A. Let the pressure ratio be $x = P_i/P_f$. Since the pressure at the bottom is greater than at the surface, $x > 1$.

$$V_B = V_i \cdot x^1 \tag{3}$$

$$V_A = V_i \cdot x^{1/\gamma} \tag{4}$$

Because $\gamma > 1$ for any gas, the exponent $1/\gamma$ is strictly less than 1. For any base $x > 1$, raising it to a smaller power yields a smaller result ($x^{1/\gamma} < x^1$).

Therefore:

$$\boxed{V_A < V_B} \tag{5}$$

Bubble B will be larger.

Physical Intuition: As Bubble A expands adiabatically, it does work against the surrounding water. Because it cannot absorb heat to compensate for this lost energy, its internal energy drops, meaning Bubble A cools down. Bubble B, on the other hand, absorbs heat from the lake to remain at T_i. Since Bubble B is warmer than Bubble A when they both reach the same pressure P_f at the surface, the Ideal Gas Law ($V \propto T$ at constant P) dictates that the warmer bubble (B) must have a larger volume.

Question 2: Diesel Engine Compression

Problem Statement: Atmospheric air is quickly compressed to about 1/20 of its original volume. Estimate the temperature of the air after compression, and explain why a Diesel engine does not require spark plugs.

Solution: Because the air is compressed "quickly", there is no time for significant heat exchange with the cylinder walls. We can treat this process as an **adiabatic compression**.

For an adiabatic process relating temperature and volume, we use the equation:

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \tag{6}$$

Solving for the final temperature T_f:

$$T_f = T_i \left(\frac{V_i}{V_f}\right)^{\gamma-1} \tag{7}$$

To estimate the numerical value, we make the following reasonable assumptions:

- The initial temperature of the atmospheric air is roughly room temperature: $T_i \approx 300$ K (approx. 27°C).
- The compression ratio is given as $V_i/V_f = 20$.
- Air is primarily composed of diatomic molecules (N_2 and O_2). For a diatomic ideal gas at room temperature, there are 5 accessible degrees of freedom (3 translational, 2 rotational), which gives an adiabatic index of $\gamma = \frac{7}{5} = 1.4$. Thus, $\gamma - 1 = 0.4$.

Substituting these values into the equation:

$$\begin{aligned} T_f &= 300 \text{ K} \cdot (20)^{0.4} \\ T_f &\approx 300 \text{ K} \cdot 3.314 \\ T_f &\approx 994 \text{ K} \end{aligned} \tag{8}$$

Converting back to Celsius, $T_f \approx 994 - 273 = 721$°C.

$$\boxed{T_f \approx 994 \text{ K} \approx 721^\circ\text{C}} \tag{9}$$

Explanation: In a standard gasoline engine, a spark plug is required to provide the activation energy to ignite the fuel-air mixture. However, in a Diesel engine, only pure air is drawn into the cylinder during the intake stroke. It is then adiabatically compressed, causing its temperature to skyrocket to approximately 700°C (or higher, depending on the exact initial temperature and pressure).

The autoignition temperature of diesel fuel is typically around 210°C. Because the compressed air in the cylinder is vastly hotter than this threshold, when the fuel is finally injected into the cylinder at the peak of compression, it **spontaneously ignites** upon contact with the superheated air. Therefore, no spark plugs are required.