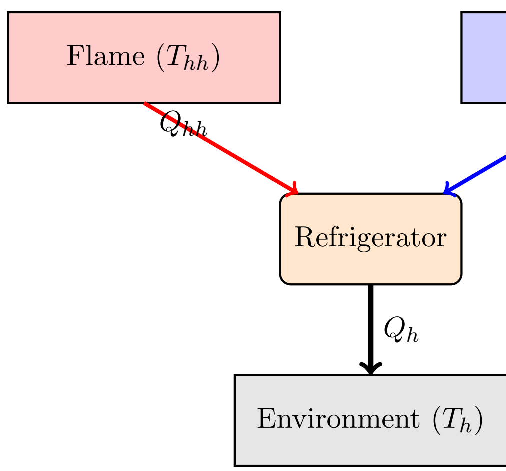
PHYS 301: Thermodynamics and Statistical Mechanics
Solutions to Problem Set #14

Question 1: Absorption Refrigerator

(a) Energy-Entropy Flow Diagram

An absorption refrigerator operates between three thermal reservoirs without any external work input ($W = 0$). Heat Q_{hh} is extracted from the high-temperature flame (T_{hh}), heat Q_l is extracted from the cold interior of the refrigerator (T_l), and the combined heat Q_h is expelled to the surrounding environment (T_h).



(b) Compute the Ratio Q_l/Q_{hh} for Reversible Operation

To solve for the heat ratio, we apply the First and Second Laws of Thermodynamics to the cyclic process of the absorption refrigerator.

1. First Law of Thermodynamics (Energy Conservation): Because the device operates in a cycle, the net change in internal energy is zero. Furthermore, no work is done ($W = 0$). Therefore, the total heat entering the system must equal the total heat leaving the system:

$$Q_h = Q_{hh} + Q_l \tag{1}$$

2. Second Law of Thermodynamics (Entropy Conservation): For a completely reversible process, the total change in entropy of the universe is zero. The entropy changes are due solely to the heat transfers at the three reservoirs:

$$\Delta S_{\text{univ}} = \frac{Q_h}{T_h} - \frac{Q_{hh}}{T_{hh}} - \frac{Q_l}{T_l} = 0 \tag{2}$$

Now, substitute Q_h from the First Law into the entropy equation:

$$\frac{Q_{hh} + Q_l}{T_h} = \frac{Q_{hh}}{T_{hh}} + \frac{Q_l}{T_l} \tag{3}$$

Group the terms involving Q_l on one side and Q_{hh} on the other:

$$\frac{Q_l}{T_h} - \frac{Q_l}{T_l} = \frac{Q_{hh}}{T_{hh}} - \frac{Q_{hh}}{T_h} \tag{4}$$

$$Q_l \left(\frac{1}{T_h} - \frac{1}{T_l} \right) = Q_{hh} \left(\frac{1}{T_{hh}} - \frac{1}{T_h} \right) \tag{5}$$

Find a common denominator for both sides:

$$Q_l \left(\frac{T_l - T_h}{T_h T_l} \right) = Q_{hh} \left(\frac{T_h - T_{hh}}{T_{hh} T_h} \right) \tag{6}$$

Multiply both sides by $-T_h$ to flip the numerators into positive temperature differences (since $T_h > T_l$ and $T_{hh} > T_h$):

$$Q_l \left(\frac{T_h - T_l}{T_l} \right) = Q_{hh} \left(\frac{T_{hh} - T_h}{T_{hh}} \right) \tag{7}$$

Finally, solve for the ratio Q_l/Q_{hh}:

$$\boxed{\frac{Q_l}{Q_{hh}} = \left(\frac{T_{hh} - T_h}{T_{hh}} \right) \left(\frac{T_l}{T_h - T_l} \right)} \tag{8}$$

Physical Intuition: Notice that this result is exactly the product of the Carnot engine efficiency (operating between T_{hh} and T_h) and the Carnot refrigerator coefficient of performance (operating between T_h and T_l). Conceptually, the heat from the flame drives a theoretical Carnot engine which then powers a theoretical Carnot refrigerator.

Question 2: Ocean Thermal Energy Conversion (OTEC)

(a) Maximum Possible Efficiency

The maximum possible efficiency of any heat engine operating between two temperatures is given by the Carnot efficiency:

$$\eta_{\text{max}} = 1 - \frac{T_C}{T_H} \tag{9}$$

First, we must convert the given Celsius temperatures to Kelvin:

$$T_H = 22^\circ\text{C} + 273.15 = 295.15 \text{ K}$$
$$T_C = 4^\circ\text{C} + 273.15 = 277.15 \text{ K}$$

Substitute these into the efficiency equation:

$$\eta_{\text{max}} = 1 - \frac{277.15}{295.15} = \frac{18}{295.15} \approx 0.0610 \tag{10}$$

$$\boxed{\eta_{\text{max}} \approx 6.1\%} \tag{11}$$

(b) Minimum Volume of Water Processed per Second

We want the engine to produce a power output of $P = 1$ GW $= 10^9$ W (Joules per second).

First, we calculate the rate at which heat $\dot{Q}_H$ must be extracted from the hot reservoir (the surface water) to produce this power, assuming the engine operates at the maximum theoretical efficiency from part (a):

$$\dot{Q}_H = \frac{P}{\eta_{\max}} = \frac{10^9 \text{ J/s}}{18/295.15} \approx 1.64 \times 10^{10} \text{ J/s} \tag{12}$$

To find the *minimum* volume of water that must be processed to provide this heat, we must extract the maximum possible energy from every drop of surface water. This happens if we extract heat until the surface water cools all the way down to the temperature of the cold reservoir ($T_C = 4°\text{C}$). *(Note: In reality, cooling the water lowers the effective T_H and thus the efficiency, but calculating the absolute lower bound on volume requires this idealized assumption).*

The heat Q extracted when a mass m of water cools by $\Delta T = 18$ K is:

$$Q = mc\Delta T = (\rho V)c\Delta T \tag{13}$$

where $\rho \approx 1000$ kg/m^3 is the density of water and $c \approx 4186$ J/(kg $\cdot$ K) is the specific heat capacity of water.

The heat extracted per unit volume of water is:

$$\frac{Q}{V} = \rho c\Delta T = (1000 \text{ kg/m}^3)(4186 \text{ J/(kg}\cdot\text{K)})(18 \text{ K}) \approx 7.53 \times 10^7 \text{ J/m}^3 \tag{14}$$

To find the required volume flow rate $\dot{V}$, we divide the required heat rate $\dot{Q}_H$ by the heat extracted per unit volume:

$$\dot{V} = \frac{\dot{Q}_H}{Q/V} = \frac{1.64 \times 10^{10} \text{ J/s}}{7.53 \times 10^7 \text{ J/m}^3} \approx 217.6 \text{ m}^3\text{/s} \tag{15}$$

$$\boxed{\dot{V} \approx 218 \text{ m}^3\text{/s}} \tag{16}$$

This is a massive flow rate—roughly equivalent to the discharge of a medium-sized river—highlighting the engineering challenge of extracting useful work from systems with such small temperature gradients.