PHYS 301
Thermodynamics and Statistical Mechanics

Problems #2

Wednesday, 01/28/2026

Question 1.

Derive an equation for the multiplicity $\Omega(N, q)$ of an Einstein solid with N oscillators and q energy quanta in the low-energy (low-temperature) limit $q \ll N$. Here, assume that N and q are both large. Start from the general multiplicity of an Einstein solid

$$\Omega(N, q) = \frac{(q + N - 1)!}{q!(N - 1)!}, \tag{1}$$

take the natural logarithm on both side, and use Stirling's approximation $\ln N! \approx N \ln N - N$ to simplify your answer.

Question 2.

Derive an expression for the multiplicity of a spin-1/2 system with N spins in the limit that $N_\downarrow \ll N$, writing your answer in terms of N and $N_\downarrow$. Remember that the general expression for the multiplicity for this system is

$$\Omega(N, N_\uparrow) = \frac{N!}{N_\uparrow! N_\downarrow!}, \tag{2}$$

with $N = N_\uparrow + N_\downarrow$. Follow a similar procedure to that of question 1. Compare your answer to what you obtain in the previous question. Do you see similarities? Why?