PHYS 301
Thermodynamics and Statistical Mechanics

Problems #3
Wednesday, 02/04/2026

Question 1.

Suppose I have an ideal gas with N particles of mass m, with total energy E and contained in a box of volume V. Compute the change of entropy as I double the volume of the box, keeping N and E constant. A reminder that the Sackur-Tetrode formula is

$$S = Nk_{\mathrm{B}}\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mE}{3Nh^2}\right)^{3/2}\right)+\frac{5}{2}\right]. \tag{1}$$

Question 2.

According to the Sackur-Tetrode equation, the entropy of a mono-atomic ideal gas can become negative when its temperature (and hence its energy) is sufficiently low. Of course this is absurd, so the Sackur-Tetrode equation must be invalid at very low temperatures. Suppose you start with a sample of helium (mass $m = 6.65 \times 10^{-27}$ Kg) at room temperature (300 K) and atmospheric pressure (1 atm $= 1 \times 10^5$ N/m^2), then lower the temperature holding the density (N/V) fixed. Pretend that the helium remains a gas and does not liquefy.

(a) First, using the formal definition of temperature

$$\frac{1}{T} = \frac{\partial S}{\partial E}, \tag{2}$$

derive the relationship between energy E and temperature T for an ideal gas (this is similar to problem 1a in Homework 2)

(b) Use this relationship to rewrite the entropy as a function of N, V, T.

(c) At what temperature would the Sackur-Tetrode equation predict that S is negative? How does this compare with the 4.1 K liquefaction temperature of helium at standard atmospheric pressure? (remember that $h = 6.626 \times 10^{-34}$ J·s and $k_{\mathrm{B}} = 1.381 \times 10^{-23}$J/K)