

PHYS 301
Thermodynamics and Statistical Mechanics

Problems #5
Wednesday, 02/18/2026

Question 1.

Consider an ultra-relativistic gas of N spinless particles obeying the energy-momentum relation $E = pc$, where c is the speed of light. (Here ultra-relativistic means that $pc \gg mc^2$, where m is the mass of the particle).

- (a) Show that the canonical partition function for such relativistic particles is given by

$$Z(V, T) = \frac{1}{N!} \left[\frac{V}{\pi^2} \left(\frac{k_B T}{\hbar c} \right)^3 \right]^N. \quad (1)$$

- (b) Use this expression to show that ultra-relativistic gas also obeys the familiar ideal gas law $PV = Nk_B T$.
- (c) Compute the average energy and heat capacity of a relativistic gas at temperature T .
- (d) Compute the entropy of this relativistic gas.