PHYS 301
Thermodynamics and Statistical Mechanics

Problems #8

Wednesday, 03/11/2026

Question 1.

So far, we have derived the equation of state for a classical monoatomic ideal gas using the canonical ensemble. Here, we will derive it using the grand canonical ensemble, where the gas is in contact with a reservoir at temperature T and chemical potential μ, and the number of particles N is allowed to fluctuate.

Recall that the canonical partition function for N indistinguishable monoatomic ideal gas particles in a volume V is:

$$Z_N = \frac{(Z_1)^N}{N!}$$

where the single-particle partition function is $Z_1 = \frac{V}{\lambda_Q^3}$, and λ_Q is the thermal de Broglie wavelength.

(a) Write down the expression for the grand canonical partition function $\mathcal{Z}$ in terms of Z_1, β, and μ. Simplify your answer into a closed-form exponential expression.
Hint: The Taylor series expansion for the exponential function is $\sum_{N=0}^{\infty} \frac{x^N}{N!} = e^x$.

(b) Calculate the Grand Canonical Potential $\Phi(T, V, \mu)$ for this ideal gas.

(c) Using the thermodynamic relations for the grand canonical potential, calculate the average number of particles $\langle N \rangle$ in the gas. Express your answer in terms of Z_1, β, and μ.

(d) In class, we established the thermodynamic relation between the grand canonical potential, pressure, and volume:

$$\Phi = -PV$$

Using this relation and your results from parts (b) and (c), compute the pressure P of the system. Show that this strictly reproduces the familiar ideal gas equation of state.

(e) Would the above equation of state change if we were to consider a diatomic ideal gas with rotational and vibrational degrees of freedom? Does it matter what the exact form of Z_1 is?