

**PHYS 301**  
**Thermodynamics and Statistical Mechanics**

Problems #9  
Wednesday, 03/25/2026

**Question 1.**

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Consider a gas of photons at temperature  $T$  in a box of volume  $V$ . Since photons are bosons, the occupancy (distribution function) of a photon with energy  $E_\gamma$  is given by the Bose-Einstein distribution

$$\langle N \rangle_{E_\gamma} = f_\gamma(E_\gamma, T) = \frac{1}{e^{E_\gamma/k_B T} - 1}, \quad (1)$$

where  $E_\gamma = \hbar\omega = pc$  is the energy of a photon with frequency  $\omega$  or momentum  $p$ . Remember that photons have two polarizations, meaning that the number of states in an infinitesimal phase space volume  $d^3x d^3p$  is

$$2 \frac{d^3x d^3p}{h^3}. \quad (2)$$

- (a) Compute the average number of photons in the box and show that it is proportional to  $T^3$ .
- (b) What is the mean energy of a photon in the box?
- (c) What is the most likely energy of a photon in the box?
- (d) Show that the partition function for this gas of photons is given by

$$\ln Z = -\frac{V}{\pi^2 c^3} \int_0^\infty d\omega \omega^2 \ln(1 - e^{-\hbar\omega/k_B T}). \quad (3)$$