PHYS 301
Thermodynamics and Statistical Mechanics

Worksheet #1

Tuesday January 27 2026

Question 1.

We saw last time that the multiplicity of a two-state spin-1/2 system with N total spins, $N_\uparrow$ up spins and $N_\downarrow = N - N_\uparrow$ down spins is

$$\Omega(N, N_\uparrow) = \frac{N!}{N_\uparrow! N_\downarrow!}. \tag{1}$$

(a) Use Stirling's formula to write the entropy of this system as

$$S = k_\mathrm{B} \left(N \ln N - N_\uparrow \ln N_\uparrow - N_\downarrow \ln N_\downarrow \right) \tag{2}$$

(b) At which value of $N_\uparrow$ is this entropy maximized? What is the value of the entropy there?