PHYS 301
Thermodynamics and Statistical Mechanics

Worksheet #10
Thursday March 12 2026

Question 1.

Fermi-Dirac distribution: Let's consider a simple system. The system can be unoccupied (with energy $E = 0$) or occupied by a single fermion (with energy $E = \epsilon$). Since the occupying particle is a fermion, no other fermion can occupying that state. The chemical potential is μ and the temperature is T.

(a) Argue that the grand partition function in this case is

$$\mathcal{Z} = \sum_n e^{-\beta(E_n - \mu N_n)} = 1 + e^{-\beta(\epsilon - \mu)}. \tag{1}$$

(b) Compute the average number of fermions in the system $\langle N \rangle_{\text{fermions}}$.

(c) This average number of particle is called the *Fermi-Dirac distribution* $f_{\text{FD}} \equiv \langle N \rangle_{\text{fermions}}$. Plot f_{FD} as a function of ϵ for both high temperatures and low temperatures. What is the occupancy of states with $\epsilon \ll \mu$? What about states with $\epsilon \gg \mu$?

Question 2.

Bose-Einstein distribution: Let's again consider a simple system. The system can be unoccupied (with energy $E = 0$), occupied by a one boson (with energy $E = \epsilon$), occupied by two bosons (with energy $E = 2\epsilon$), occupied by three bosons (with energy $E = 3\epsilon$), and so on. The chemical potential is μ and the temperature is T.

(a) Assuming $\mu < \epsilon$, compute the grand partition function for this system

$$\mathcal{Z} = \sum_n e^{-\beta(E_n - \mu N_n)}. \tag{2}$$

Remember that

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad \text{for} \qquad |x| \ll 1. \tag{3}$$

(b) Compute the average number of bosons in the system $\langle N \rangle_{\text{bosons}}$.

(c) This average number of particle is called the *Bose-Einstein distribution* $f_{\text{BE}} \equiv \langle N \rangle_{\text{bosons}}$. Plot f_{BE} as a function of ϵ, assuming $\epsilon > \mu$. What is the occupancy of states with $\epsilon \sim \mu$?