PHYS 301
Thermodynamics and Statistical Mechanics

Worksheet #13
Thursday April 2 2026

Question 1.

Even at zero temperature, fermions can have significant energy since they need to fill states all the way to the Fermi surface because of the Pauli exclusion principle. This implies that even cold fermions can have significant pressure. Under the right conditions, this pressure can be so great as to hold an entire star against gravitational collapse. Let's compute this so-called *degeneracy pressure* for spin-1/2 particles.

(a) First, let's compute the average energy $\langle E \rangle$ of a cold ($T \to 0$) fermion gas. To begin, justify that $\langle E \rangle$ can be approximated as

$$\langle E \rangle = \sum_r \frac{E_r}{e^{\beta(E_r - \mu)} + 1} = \int_0^\infty dE \frac{E\, g(E)}{e^{\beta(E-\mu)} + 1} \approx \int_0^{E_F} dE\, E\, g(E), \tag{1}$$

(where $g(E) \propto \sqrt{E}$ is the density of states) and then compute $\langle E \rangle$ as a function of E_F.

(b) Use the relationship

$$P = -\left.\frac{\partial \langle E \rangle}{\partial V}\right|_{S,N} \tag{2}$$

to compute the degeneracy pressure of a cold fermion gas. Express your answer in terms of E_F.

(c) The Fermi temperature for conduction electrons inside a block of copper is about $T_F = E_F/k_\mathrm{B} = 8.15 \times 10^4$ K. If their density is $N/V = 8.46 \times 10^{28}\ \mathrm{m}^{-3}$, show that the degeneracy pressure of these electrons is at least *5 orders of magnitude larger* than standard atmospheric pressure (1 atm $\sim 10^5\ \mathrm{N/m}^2$). Degeneracy pressure is not small!