PHYS 301
Thermodynamics and Statistical Mechanics

Worksheet #16

Tuesday April 21 2026

Question 1.

Consider an ideal monoatomic gas with N particles in an initial volume V_i and initially at temperature T_i. We want to compress this gas quasi-statically to a final volume $V_f < V_i$. We consider here two different ways if doing so:

(a) Isothermal compression: In this case, we compress the gas slowly such that its temperature stays constant. While we are clearly doing work on the gas, its total energy ($E = (3/2)Nk_\mathrm{B}T_i$) is unchanged. Use the first law of thermodynamics to show that the heat escaping the system must be

$$Q = Nk_\mathrm{B}T_i \ln\left(\frac{V_f}{V_i}\right). \tag{1}$$

(b) Adiabatic compression: In this case, we isolate the system such that no heat flows in or out of the system (i.e. $Q = 0$), and the work done will increase the total energy of the gas. Since $E = (3/2)Nk_\mathrm{B}T$, this means that the gas will heat up (to $T = T_f > T_i$) as we compress it. Use the first law of thermodynamics (first relate dE to dT) to show that this increase in temperature is governed by the following equation

$$V_f T_f^{3/2} = V_i T_i^{3/2}. \tag{2}$$

Th implies that adiabatic compressions (or expansions) of ideal monoatomic gases obey $VT^{3/2} = \text{constant}$. Use the ideal gas law to show that this implies

$$PV^\gamma = \text{constant}, \tag{3}$$

where γ is called the *adiabatic index* of the gas (which is equal to 5/3 for a monoatomic gas).