PHYS 301
Thermodynamics and Statistical Mechanics

Worksheet #18
Tuesday April 28 2026

Question 1.

Ideal gas Carnot Engine: Consider a Carnot engine working with a monoatomic ideal gas between a high temperature $T_{\rm H}$ and and a low temperature $T_{\rm C}$. As usual, the Carnot cycle has 4 parts (see figure): an isothermal expansion at $T = T_{\rm H}$ (AB), an adiabatic expansion (BC), an isothermal compression at $T = T_{\rm C}$ (CD), and an adiabatic compression (DA). Remember that the energy of a monoatomic ideal gas is $E = (3/2)Nk_{\rm B}T$.

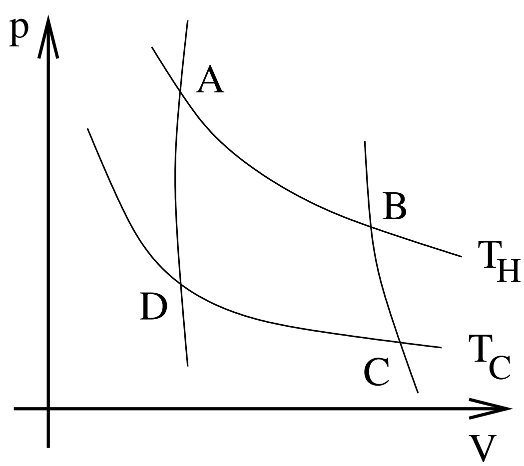

Figure 1: A Carnot Engine

(a) Compute the heat absorbed $Q_{\rm H}$ during the isothermal expansion (AB). Express your answer in terms of $T_{\rm H}$, V_B, and V_A.

(b) Compute the heat released $Q_{\rm C}$ during the isothermal compression (CD). Express your answer in terms of $T_{\rm C}$, V_C, and V_D.

(c) For the adiabatic parts of the cycle (BC and DA), show that $VT^{3/2}$ = constant. Use this to show that

$$\frac{V_A}{V_B} = \frac{V_D}{V_C}. \tag{1}$$

(d) Use the above to show that the Carnot efficiency is simply

$$\eta_{\rm C} = 1 - \frac{Q_{\rm C}}{Q_{\rm H}} = 1 - \frac{T_{\rm C}}{T_{\rm H}}. \tag{2}$$

This defines the *thermodynamic temperature.*

(e) The above implies that

$$\frac{Q_\mathrm{H}}{T_\mathrm{H}} - \frac{Q_\mathrm{C}}{T_\mathrm{C}} = 0 \tag{3}$$

Adopting the notation that heat *leaving* the system is negative, the above could be written as

$$\sum_{i=1}^{2} \frac{Q_i}{T_i} = 0, \tag{4}$$

where $Q_1 = Q_\mathrm{H}$ with $T_1 = T_\mathrm{H}$, and $Q_2 = -Q_\mathrm{C}$ with $T_2 = T_\mathrm{C}$. The above looks like a conservation law for reversible cycles. What is being conserved here? Look at the units for guidance and remember that the above is a consequence of the second law.

(f) Argue that any reversible cycle can be broken down into N isothermal processes and N adiabatic processes, as illustrated in the figure below. Generalize the above to show that, for any reversible cycle, we have

$$\oint_R \frac{đQ}{T} = 0, \tag{5}$$

where the subscript R means reversible. This means that whatever quantity you found was conserved in part (e) is a function of state. What is this function of state?

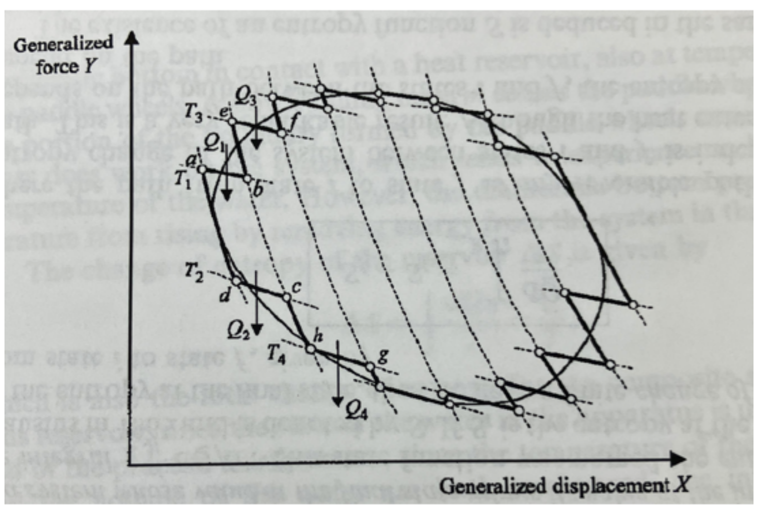

Figure 2: Any reversible cycle can be broken down into a series of isothermal and adiabatic processes.