

PHYS 301

Thermodynamics and Statistical Mechanics

Worksheet #3
Tuesday February 10 2026

Question 1.

Consider the setup we just discussed in class in which we have a large reservoir R at temperature T in thermal contact with a system S . We denote the microstates of S by $|n\rangle$, each with energy E_n . The combination $R + S$ has total energy $E_{\text{tot}} \gg E_n$. Here we want to derive the probability $p(n)$ of finding the system S in a specific microstate $|n\rangle$. Do this problem in team at the whiteboard.

- (a) As we just saw, the total number of microstates of the combined $R + S$ systems is

$$\Omega(E_{\text{tot}}) = \sum_n \Omega_R(E_{\text{tot}} - E_n) \cdot 1 = \sum_n \exp\left[\frac{S_R(E_{\text{tot}} - E_n)}{k_B}\right], \quad (1)$$

where the sum runs over all possible microstates $|n\rangle$ of S (not the energy levels of S). In the limit that $E_n \ll E_{\text{tot}}$, approximate the above expression such that it factors out into two pieces: one that only depends on the reservoir R , and another that depends only on the system S and the temperature T .

- (b) For each microstate $|n\rangle$ of the system S , how many total states of the combined $S + R$ systems are there?
- (c) Now apply the fundamental assumption of statistical mechanics to the states of the combined $S + R$ systems to derive the probability $p(n)$ of finding the system S in a specific microstate $|n\rangle$. Make sure your probability distribution is properly normalized (i.e. $\sum_n p(n) = 1$). Do the details of the large reservoir enter this probability distribution (except through T)? Should it?
- (d) Now half your team moves (clockwise) to the next team to look and discuss their results. Do their answers match yours? To ensure that their answers make sense, go through the following check list:
- Is the probability distribution $p(n)$ properly normalized?
 - Are microstates with $E_n \gg k_B T$ likely to be populated? If yes, is that strange?
 - How about states with $E_n \leq k_B T$? Can they be populated? Should they?
 - Assuming that the system S has a lowest energy state with $E_n = 0$, does the $T \rightarrow 0$ limit make sense?
- (e) Now return to your original team, and correct your answer if necessary. You have just derived one of the most useful results in all of physics.