PHYS 301
Thermodynamics and Statistical Mechanics

Worksheet #4

Tuesday February 17 2026

Question 1.

The equipartition theorem: For system in of N particle in equilibrium at some temperature T, we often say that each "degree of freedom" (e.g. translation, rotation, vibration) has $(1/2)Nk_{\mathrm{B}}T$ of energy. We will see that this works only for "quadratic" degrees of freedom, that is, those with energy of the form

$$E(q) = c\,q^2, \tag{1}$$

where q can be a spatial coordinate, momentum, angular momentum, etc., and c is a constant that takes care of the units.

(a) Compute the partition function for one particle with the above energy. Assume that the different q states are separated by a very small Δq, allowing us to write the partition function as

$$Z_{1\,\mathrm{part}} = \sum_q e^{-\beta E(q)} = \frac{1}{\Delta q}\sum_q e^{-\beta E(q)}\Delta q. \tag{2}$$

Approximate the sum above as an integral and perform it. It's fine to have Δq in the prefactor.

(b) Now, assuming that the N particles are indistinguishable, write the partition function for N particles.

(c) Compute the average energy $\langle E\rangle$ of this system. Note that the answer does not depend on the details of the system (that is, the answer does not depend on c and Δq), showing that this is true for any system with energy depending on quadratic variables. This shows that every "quadratic" degree of freedom does has $(1/2)k_{\mathrm{B}}T$ of energy per particle.