

PHYS 301
Thermodynamics and Statistical Mechanics

Worksheet #8
Thursday March 5 2026

Question 1.

Carbon Monoxide Poisoning: Oxygen is carried in the blood by hemoglobin. This molecule has four O_2 absorption sites, each consisting of an Fe^{2+} ion that can bind with oxygen. Here, let's focus on one of these absorption sites. Assuming for now that O_2 is the only molecule that can occupy the site, the system has just two possible states: unoccupied (with energy 0) and occupied (with binding energy $B_{O_2} = -0.7$ eV).

- (a) Write down the grand partition function \mathcal{Z} for this single-site system, assuming that O_2 has some chemical potential μ_{O_2} .
- (b) The chemical potential μ_{O_2} varies significantly in different parts of the body. Here, let's focus on the lungs, where oxygen is abundant. There the blood is in approximate *diffusive equilibrium* with the atmosphere. Taking the atmosphere to be an ideal gas in which the partial pressure of oxygen is about 0.2 atm, compute the chemical potential μ_{O_2} in eV at body temperature ($T = 310$ K). Remember the expression for the chemical potential of a diatomic molecule you derived last time:

$$\mu_{\text{dia}} = -k_B T \ln \left(\frac{V Z_{\text{int}}}{N \lambda_Q^3} \right), \quad \text{with} \quad \lambda_Q \equiv \frac{h}{\sqrt{2\pi m k_B T}}, \quad (1)$$

and where $Z_{\text{int}} = Z_{\text{rot}} Z_e$ (last time, you did this calculation with $Z_{\text{int}} = Z_{\text{rot}}$ only, but here we need to take the electronic degeneracy Z_e too). Here, $\lambda_Q \approx 1.8 \times 10^{-11}$ m. The rotational energy constant for O_2 is $\epsilon_{O_2} = 1.8 \times 10^{-4}$ eV and its electronic degeneracy constant is $Z_e = 3$. Use the ideal gas law to relate N and V to P and T . $1 \text{ atm} \simeq 10^5 \text{ N/m}^2$.

- (c) Compute the probability that an hemoglobin site is occupied by an O_2 molecule in the lungs.
- (d) Now assume that there is also some carbon monoxide (CO) present in the lungs, which can also occupy one of the Fe^{2+} sites on hemoglobin with binding energy B_{CO} . This means that we now have three available states for our single-site system: unoccupied, occupied by O_2 , and occupied by CO. Assuming that CO has some chemical potential μ_{CO} , write down the grand partition function in this case.
- (e) Assuming that CO is 100 times *less* abundant than O_2 in the lungs, compute the chemical potential of CO there. To a good approximation, λ_Q for CO is the same as for O_2 above. The rotational energy constant is $\epsilon_{CO} = 2.4 \times 10^{-4}$ eV and $Z_e = 1$ for carbon monoxide.
- (f) Using the fact that CO has a binding energy to hemoglobin of $B_{CO} = -0.85$ eV and the chemical potentials computed above, compute the probability that an hemoglobin site is occupied by O_2 when a small amount of CO is present. What is the probability that the site

is occupied by CO? Use this to discuss why even a tiny amount of carbon monoxide in the lungs can be deadly.