PHYS480/581 Cosmology
Big-Bang Nucleosynthesis: Helium abundance

(Dated: October 31, 2022)

I. DEUTERIUM FORMATION

Last time we saw that the fractional neutron abundance $X_n(t)$ after neutron freeze-out is given by

$$X_n(t) \simeq \frac{1}{6} e^{-t/\tau_n} \tag{1}$$

where $\tau_\mathrm{n} = 886.7 \pm 0.8$ sec is the neutron lifetime. The neutron abundance slowly decays away until nucleosynthesis starts and these neutrons become bound into higher mass nuclei. With its large binding energy per nucleon, one might think that all available neutrons would rapidly assemble themselves into ^{4}He nuclei. However, for $T < 1$ MeV the matter density is just too low for three or more nucleons to come together and form heavier nuclei. Instead, helium production occurs first by assembling simpler nuclei in two-particle reactions, and then fusing these nuclei. The simplest such nucleus that first forms is deuterium, which can be produced via the reaction

$$n + p \leftrightarrow \mathrm{D} + \gamma. \tag{2}$$

We computed last week that the equilibrium deuterium abundance

$$\left(\frac{n_\mathrm{D}}{n_p}\right)_\mathrm{eq} \approx 4\eta_\mathrm{b} \left(\frac{T}{m_p}\right)^{3/2} e^{B_\mathrm{D}/T}, \tag{3}$$

where $\eta_\mathrm{b} \sim 6 \times 10^{-10}$ is the baryon to photon ratio, and $B_\mathrm{D} = 2.22$ MeV is the binding energy of deuterium. This means that the deuterium abundance is *very small* until $T \ll B_\mathrm{D}$ and the exponential factor can start compensating for the smallnest of η_b. Since nucleosynthesis cannot start until deuterium forms, let's determine the temperature T_nuc at which $n_\mathrm{D}/n_\mathrm{p} \sim 1$. Setting the left-hand side of Eq. (3) to unity and taking the natural logarithm on both sides we get

$$0 = \ln\left(4\eta_\mathrm{b}\right) + \frac{3}{2}\ln\left(\frac{T_\mathrm{nuc}}{m_\mathrm{p}}\right) + \frac{B_\mathrm{D}}{T_\mathrm{nuc}}. \tag{4}$$

This is an implicit equation for the temperature T, which we can solve iteratively by first writing

$$T_\mathrm{nuc} \approx \frac{-B_\mathrm{D}}{\ln\left(4\eta_\mathrm{b}\right)} \approx 0.112\,\mathrm{MeV}, \tag{5}$$

and then substituting this first guess into the full expression

$$T_\mathrm{nuc} \approx \frac{-B_\mathrm{D}}{\ln\left(4\eta_\mathrm{b}\right) + \frac{3}{2}\ln\left(\frac{0.112\,\mathrm{MeV}}{m_\mathrm{p}}\right)} \approx 0.066\,\mathrm{MeV}. \tag{6}$$

Repeating this exercise a few more times, we obtain $T_\mathrm{nuc} \simeq 0.065$ MeV, more than an order of magnitude smaller than the neutron freeze-out temperature. When this temperature is reached, nucleosynthesis starts in earnest, and nearly all available neutrons combine with a proton to form deuterium. But how many protons are available at $T_\mathrm{nuc} \simeq 0.065$ MeV? To determine this, we need to know the age of the Universe at this temperature.

II. TIME-TEMPERATURE RELATION

We know that the age of the Universe at a given scale factor is

$$t(a) = \int_0^a \frac{da'}{a' H(a')}, \tag{7}$$

where H is computed from the Friedmann equation in the radiation-dominated era

$$H = \sqrt{\frac{8\pi G}{3}\rho_{\rm rad}}, \qquad \text{where} \qquad \rho_{\rm rad} = \frac{\pi^2}{30} g_*(T) T^4. \tag{8}$$

The problem is that we know $\rho_{\rm rad}$ in terms of T and not a. When $g_*(T)$ (or $g_{*S}(T)$) is constant, we already know that $T \propto 1/a$, but when it is changing, it is no longer true. In general, one would need to consider the conservation of entropy $g_{*S}(T)T^3a^3$ =constant to solve for $T(a)$. However, after e^+e^- annihilation $g_*(T)$ and $g_{*S}(T)$ are constant and we have $T \propto 1/a$, implying that we have

$$\frac{da}{a} = -\frac{dT}{T} \tag{9}$$

Taking $g_*(T)$ to be constant, we thus have

$$t(T) \approx 3\sqrt{\frac{5}{4\pi^3 G}} \frac{1}{\sqrt{g_*(T)}} \int_T^\infty \frac{dT'}{T'^3} = \frac{3}{2}\sqrt{\frac{5}{4\pi^3 G}} \frac{1}{\sqrt{g_*(T)}} T^{-2}. \tag{10}$$

Putting the units back in, we get the approximate relation (see HW4)

$$\frac{t}{\rm sec} \simeq \frac{2.42}{\sqrt{g_*}} \left(\frac{T}{\rm MeV}\right)^{-2}. \tag{11}$$

So the age of the Universe at the onset of nucleosynthesis when $T_{\rm nuc} \simeq 0.065$ MeV and $g_*(T_{\rm nuc}) = 3.38$ is

$$t_{\rm nuc} \simeq \frac{2.42}{\sqrt{3.38}} \left(\frac{0.065}{\rm MeV}\right)^{-2} \text{sec} \simeq 311\,\text{sec}, \tag{12}$$

which is about 5 minutes after the Big Bang. The fact that we have to wait so long after neutron freeze-out (which occurs at $t \sim 1$ sec) to start forming composite nuclei is called the *deuterium bottleneck*.

III. HELIUM ABUNDANCE

Once deuterium starts forming, it is possible to start forming higher-mass nuclei. However, by then the expansion of the Universe has diluted the volumetric abundance of baryons so much that only helium can form with any appreciable abundance. In particular, ^{4}He can be assembled via the reactions

$$\mathrm{D} + p \leftrightarrow {}^3\mathrm{He} + \gamma, \tag{13}$$

$$\mathrm{D} + {}^3\mathrm{He} \leftrightarrow {}^4\mathrm{He} + p. \tag{14}$$

^{4}He has such a strong binding energy per nucleon that is energetically favorable for nearly all the deuterium and ^{3}He available to be burned into ^{4}He. The abundance of neutrons at $t_{\rm nuc}$ is

$$X_n(t_{\rm nuc}) \simeq \frac{1}{6} e^{-t_{\rm nuc}/\tau_n} \approx 0.12. \tag{15}$$

If all these neutrons are processed into ^{4}He, we then have $n_{\rm He}(t_{\rm nuc}) = \frac{1}{2} n_n(t_{\rm nuc})$ since two neutrons go into each ^{4}He nucleus. The ratio of ^{4}He to hydrogen is then given by

$$\frac{n_{\rm He}}{n_{\rm H}} = \frac{n_{\rm He}}{n_{\rm p}} \simeq \frac{\frac{1}{2}X_n(t_{\rm nuc})}{1 - X_n(t_{\rm nuc})} \simeq \frac{1}{2} X_n(t_{\rm nuc}) \simeq 0.06. \tag{16}$$

We usually like to express this result in terms of the *mass fraction* in helium

$$Y_{\rm P} \equiv \frac{4n_{\rm He}}{n_{\rm H}} \simeq 0.24. \tag{17}$$

Thus about a quarter of the baryonic mass ends up in helium nuclei, with the rest being hydrogen. Trace amounts of unburned D and ^{3}He are also left behind. Small amount of ^{7}Be and ^{7}Li are also produced, with extremely small abundances. See figures below.

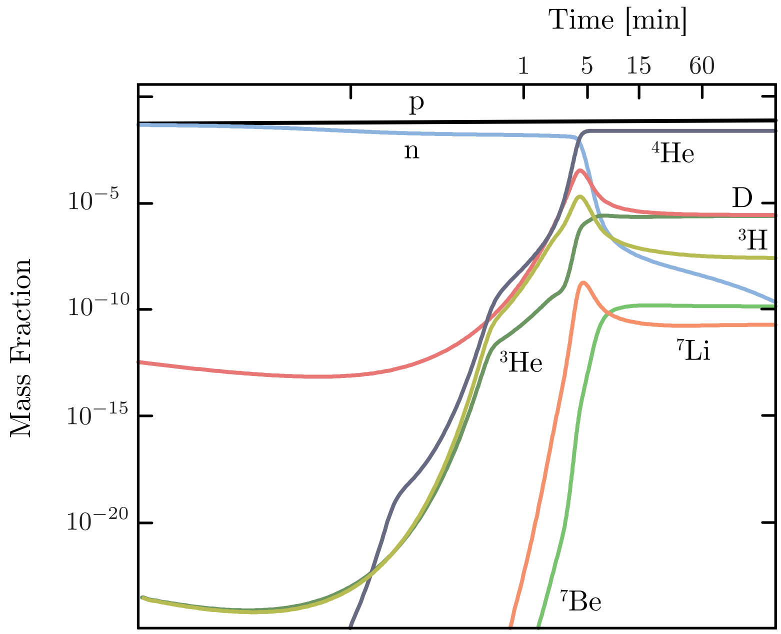

FIG. 1. Detailed evolution of the abundances of light elements during Big Bang Nucleosynthesis. Figure from Baumann (2022)

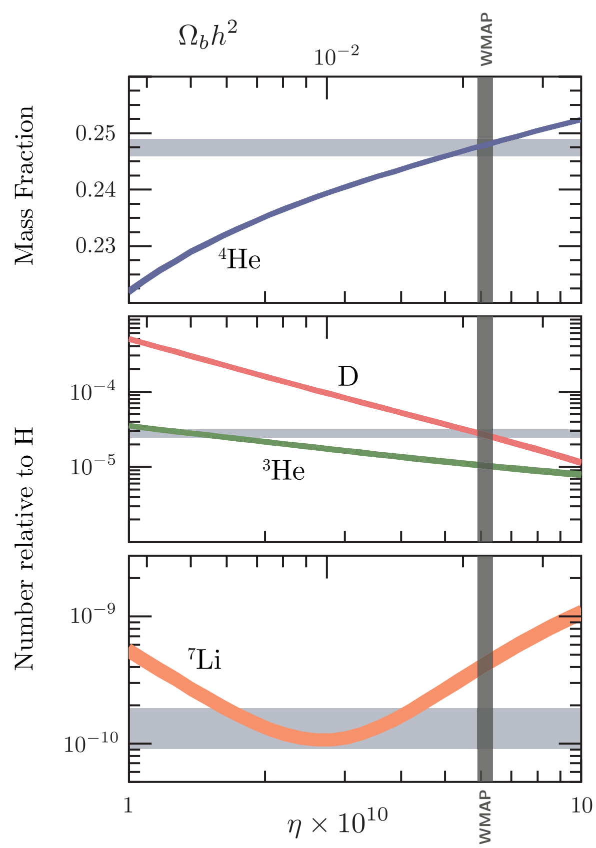

FIG. 2. Comparison between the theoretical predictions and the measured abundances of light elements. Figure from Baumann (2022)