PHYS480/581 Cosmology
Motivation for Inflation

(Dated: November 14, 2022)

I. PROBLEMS WITH THE HOT BIG BANG

As we discussed last week, once neutral atoms form during the epoch of recombination, the photons cease to interact with the baryonic matter and start to propagate freely through the Universe. We observe these photons today as the cosmic microwave background (CMB). As I have mentioned, the CMB appears isotropic across the night sky with a mean temperature today of $T_0 = 2.7255$ K.

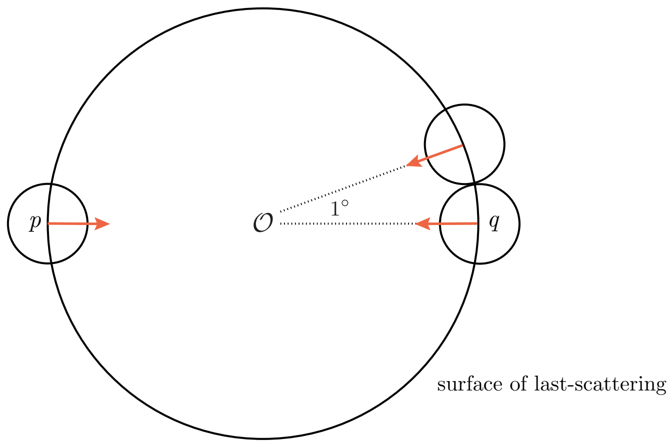

FIG. 1. Causal structure of the last-scattering surface. The small circles represent the largest distance that information could have travelled between the Big Bang and the epoch of last scattering. The large circle represents the last-scattering surface where all the CMB photons we observe today were emitted. Clearly, multiple regions of the last-scattering surface were not in causal contact with each other at that time. Figure from Baumann (2022).

A. The horizon problem

The apparent isotropy of the CMB poses an immediate problem: how can two photons coming from opposite directions in the sky have the same temperature? These photons could not have been in thermal contact since they were emitted from two different points very distant from each other at the epoch of last scattering (see points p and q in Fig. 1 above). But the problem is even worse than that: even nearby points on the sky could not have been in thermal contact at the epoch of last scattering. To see this, consider the comoving distance that a photon could have travelled from the Big Bang to the epoch of last scattering

$$\eta_* = \int_{z_*}^{\infty} \frac{dz}{H(z)}, \tag{1}$$

where z_* is the redshift of photon last-scattering. We know however that photons cannot travel very far in the early Universe since they keep scattering off free electrons. In fact, until z_* the mean free path of photons is very small and thus sending photons is not a very efficient way to transmit information on large distances. As long as the Universe is ionized, a much better way to send signal over long distances is to send plasma *sound waves*. These waves travel at the speed of sound (rather than the speed of light). This sound speed $c_{\rm s}$ can be estimated in the radiation-dominated epoch of the early Universe

$$c_{\rm s}^2 \equiv \frac{\dot{P}}{\dot{\rho}} = \frac{\frac{1}{3}\dot{\rho}}{\dot{\rho}} = \frac{1}{3}. \tag{2}$$

Thus, a better estimate for the comoving distance that *information* could have travelled between the Big Bang and last scattering is

$$\eta_* = \int_{z_*}^{\infty} \frac{c_\mathrm{s} dz}{H(z)} \simeq 161\,\mathrm{Mpc}, \tag{3}$$

where we have used $\Omega_\mathrm{m} = 0.311$, $\Omega_\mathrm{rad} = 9.1 \times 10^{-5}$, $\Omega_K = 0$, and $H_0 = 67.66$ km/s/Mpc. Meanwhile, the comoving distance travelled by a photon after last scattering to get to us is

$$\chi_* = \int_{0}^{z_*} \frac{dz}{H(z)} \simeq 13,880\,\mathrm{Mpc}. \tag{4}$$

This implies that the maximum angle on the sky between CMB photons that were in causal contact is

$$\theta = \frac{2\eta_*}{\chi*} \simeq 1.3\,\mathrm{degrees}, \tag{5}$$

which is about twice the size of the full Moon on the night sky. CMB photons arriving to us separated by larger angles were never in causal contact in the past. How can they have the same temperature then?? This is a serious problem for cosmology. But it gets worse than that. The CoBE satellite discovered in the 1990s that the CMB is not *exactly* isotropic, but instead contains small fluctuations at the level of 1 part in 10 thousands. The surprise is that some of these fluctuations are spatially very extended, spanning large regions of the sky as can be seen from Fig. 2. We say that these fluctuations must have been caused by *super-horizon modes*, since only physics acting beyond the causal horizon could have caused such perturbations.

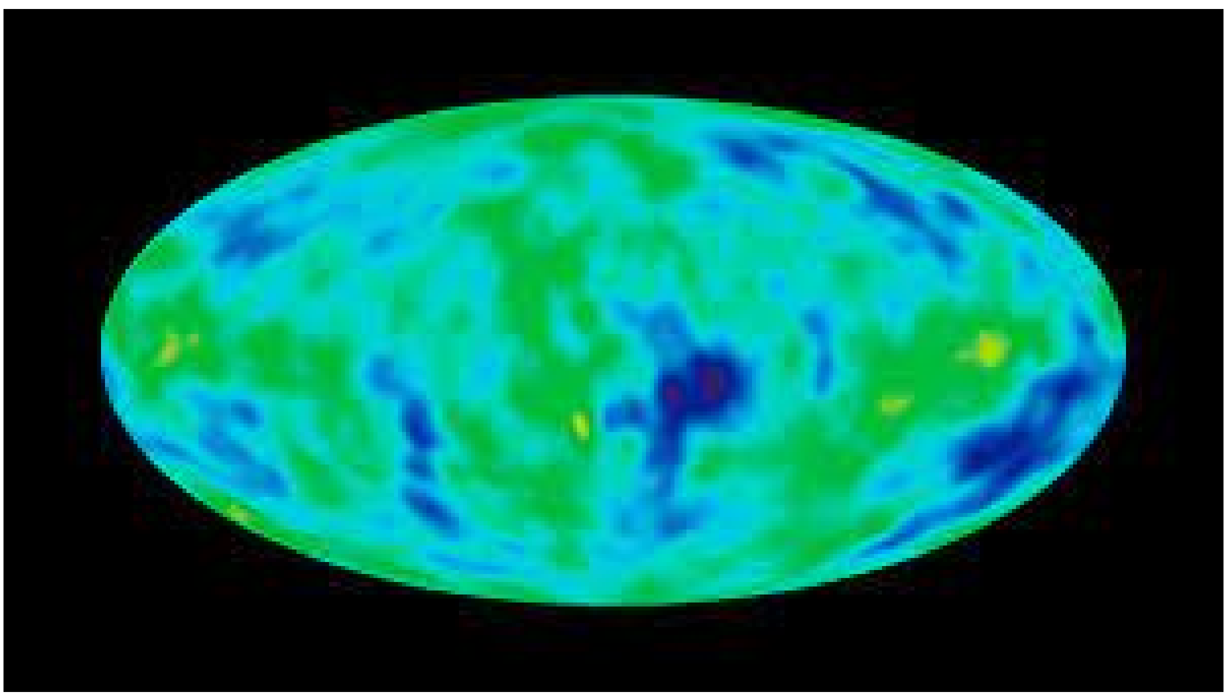

FIG. 2. The cosmic microwave background as observed by the CoBE satellite in the early 1990s. Here we are showing the photon temperature contrast $\Delta T/T$. We observe temperature fluctuations spanning large areas of the sky.

B. The flatness problem

According to current measurements, the Universe appears to be spatially flat $\Omega_K \approx 0$. This is slightly surprising since we know that curvature "redshifts" slower ($\rho_K \propto (1+z)^2$) than either radiation ($\rho_\mathrm{rad} \propto (1+z)^4$) or matter ($\rho_\mathrm{m} \propto (1+z)^3$). We would thus naturally expect curvature to start dominating over matter at some point in the history of the Universe. But this clearly didn't happen. To put this problem in a broader context, consider the Friedmann equation

$$H^2 = \frac{8\pi G}{3}\rho_{\rm tot} - \frac{k}{a^2}, \tag{6}$$

which can be re-written as

$$\frac{|k|}{a^2 H^2} = \left|\frac{8\pi G}{3H^2}\rho_{\rm tot} - 1\right|, \tag{7}$$

where we have taken absolute value since k could be negative. Now, in matter domination we have $H^2 \propto a^{-3}$, while in radiation domination we have $H^2 \propto a^{-4}$. The above can then be rewritten as

$$\left|\frac{8\pi G}{3H^2}\rho_{\rm tot} - 1\right| \propto \begin{cases} |k|a & \text{if matter dominates,} \\ |k|a^2 & \text{if radiation dominates.} \end{cases} \tag{8}$$

We thus see that, unless $k = 0$ identically, the left-hand side is a growing function of time. That is, a flat Universe ($k = 0$) is an unstable configuration; any perturbation away from flatness will get amplified over time. This points to extremely stringent constraints on k since to infer $k \sim 0$ today (that is, after the Universe has expanded by a very large factor), k must have been insanely close to zero in the early Universe. We call this the flatness problem.

To get a quantitative estimate of how bad this problem is, let's estimate how fine-tuned k needs to be match current observations. The latest measurements from the Planck satellite, aided by some large-scale structure data, constrain the curvature to be $\Omega_K < 0.001$. Since $\Omega_K = -k/H_0^2$, this tells us that $|k|/H_0^2 < 0.001$ today. At the epoch of last scattering ($a \sim 10^{-3}$), we must have had

$$\frac{|k|}{H^2} \lesssim 10^{-9} \qquad \text{at last scattering,} \tag{9}$$

while at the onset of Big Bang Nucleosynthesis ($a \sim 10^{-10}$), we must have had

$$\frac{|k|}{H^2} \lesssim 10^{-23} \qquad \text{at BBN,} \tag{10}$$

and finally during electroweak symmetry breaking ($a \sim 10^{-15}$), we must have had

$$\frac{|k|}{H^2} \lesssim 10^{-33} \qquad \text{at electroweak breaking.} \tag{11}$$

This is an extreme level of tuning to have without a clear physical reason why. For a different perspective on this problem, remember that the curvature term can be written as $k = \kappa/R^2$, where $\kappa = \{-1, 0, 1\}$, and R is the radius of curvature of the Universe. This last tuning can then be written as

$$|R| \gtrsim 10^{33} H^{-1}. \tag{12}$$

Remember that H^{-1} is the Hubble radius which sets the size of the observable Universe at any given time. The puzzle here is why was the radius of curvature 33 orders of magnitude larger than the typical size of the Universe at that time. Why this huge hierarchy? Physicists don't like having numbers that are vastly different entering the same problem without good reasons.

II. INFLATION

In 1981, Alan Guth came up with a solution to address both the horizon and flatness problem: inflation. In its simplest form, inflation is a period of accelerated expansion in the very early Universe, that is a period in which $\ddot{a}(t) > 0$. From the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + 3P\right), \tag{13}$$

having $\ddot{a}(t) > 0$ requires

$$P < -\frac{\rho}{3}. \tag{14}$$

We know a form of energy that can achieve this: a cosmological constant. Thus is the Universe was dominated at very early times by a nearly constant energy density with $w \approx -1$, the scale factor would admit an exponential form

$$a(t) \propto e^{H_I t}, \tag{15}$$

where H_I is the (constant) Hubble rate during inflation. Since H_I is (nearly) constant during inflation, then the size of the Hubble radius is constant, while the size of the Universe as a whole is made exponentially larger.