PHYS480/581 Cosmology
Inflationary dynamics and reheating

(Dated: November 16, 2022)

I. INFLATION AS A SOLUTION TO THE HORIZON AND FLATNESS PROBLEMS

Last time, we discussed the horizon and flatness problems of the hot Big Bang model.

1. **Horizon:** Two points on the CMB sky separated by more than ~ 1 degree cannot have been in causal contact in the past, yet these CMB photons have the same temperature to a very good approximation.

2. **Flatness:** The deviation from flatness at any given time t can be written as

$$|\Omega_{\rm tot}(t) - 1| = \frac{|k|}{a^2 H^2}, \qquad \text{with} \qquad \Omega_{\rm tot}(t) = \frac{8\pi G \rho_{\rm tot}(t)}{3H^2(t)}. \tag{1}$$

   Since $H^2 \propto a^{-4}$ in radiation domination and $H^2 \propto a^{-3}$ in matter domination, the right-hand side is always a growing function of the scale factor when $|k| \neq 0$. Yet, we measure today that $|\Omega_{\rm tot}(t_0) - 1| < 0.001$, which means that $|k|/H^2$ must have been *extremely small* at early times.

Both of these problems can be solved by postulating a brief period of *inflation*, a period of accelerated expansion with $\ddot{a}(t) > 0$. As we saw last time, this requires negative pressure $P < -\rho/3$. Possibly, the simplest way of achieving this is to have the Universe dominated by a constant energy density which doesn't dilute with the expansion. Such model has $P = -\rho$ and the scale factor for such model is simply

$$a(t) \propto e^{H_I t}, \tag{2}$$

where H_I is the Hubble expansion rate during inflation, which is a constant (since the energy density is a constant). Such exponential expansion can stretch the causal horizon so much that all parts of the CMB last-scattering surface were in causal contact in the past. This is illustrate in Fig. 1 below.

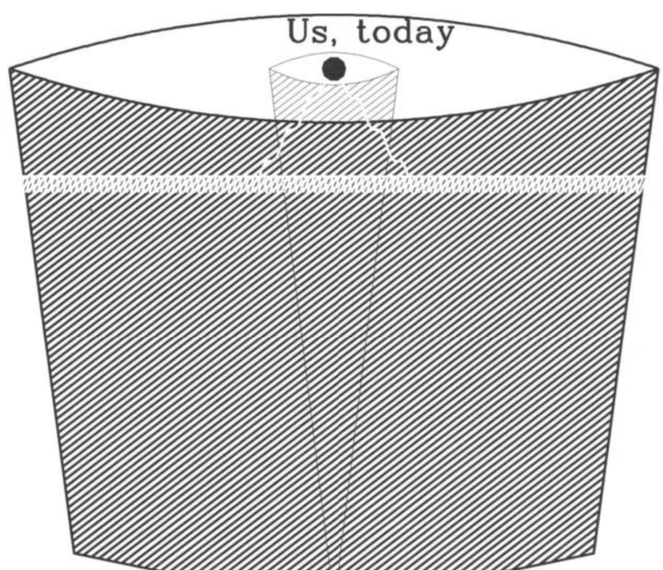

FIG. 1. Inflationary solution to the horizon problem. Larger cone shows the true horizon in an inflationary model; smaller inner cone shows the horizon without inflation. During inflation, the physical horizon blows up very rapidly. All scales in the shaded region were once in causal contact so it is not surprising that the temperature is uniform. Figure from Dodelson (2003).

Concerning the flatness problem, a period with $H_I \sim$ constant and $a(t) \propto e^{H_I t}$ leads to

$$|\Omega_{\rm tot}(t) - 1| \propto e^{-2H_I t}, \tag{3}$$

that is, the difference from flatness decays exponentially fast during inflation. This sets its initial value so small that all the subsequent expansion of the Universe cannot really make it big again. This solves the fine-tuning problem. Another way to think about this is that inflation makes the entire Universe so large that it's not a surprise that today the radius of curvature of the Universe is $R \gg H_0^{-1}$.

II. SIMPLE MODELS OF INFLATION

The simplest models of inflation involves scalar fields. A scalar field is simply a function of space and time $\phi(\mathbf{r}, t)$. We call them "scalar" since they are basically just numbers at every point in spacetime, as opposed to, say, a vector field (like the electric or magnetic fields) which is a vector at every point in spacetime. Just like electric fields, scalar fields can carry energy and momentum and have dynamical evolution. In particular, the energy density in a scalar field $\phi(\mathbf{r}, t)$ is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{4}$$

where $\dot{\phi} = d\phi/dt$. The first term represents the kinetic energy of the field, while $V(\phi)$ is the potential energy of the field. Meanwhile, the pressure of a scalar field is

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{5}$$

This means that if the energy of the field is dominated by the potential energy, the equation of state of the scalar field would be

$$w_\phi = \frac{P_\phi}{\rho_\phi} \approx \frac{-V(\phi)}{V(\phi)} = -1, \tag{6}$$

that is, this field is behaving approximately like a cosmological constant and can thus drive inflation. This is the basis for a lot of the inflation literature: it needs a field that moves slowly ("slow-roll") such that $V(\phi) \gg \dot{\phi}^2$.

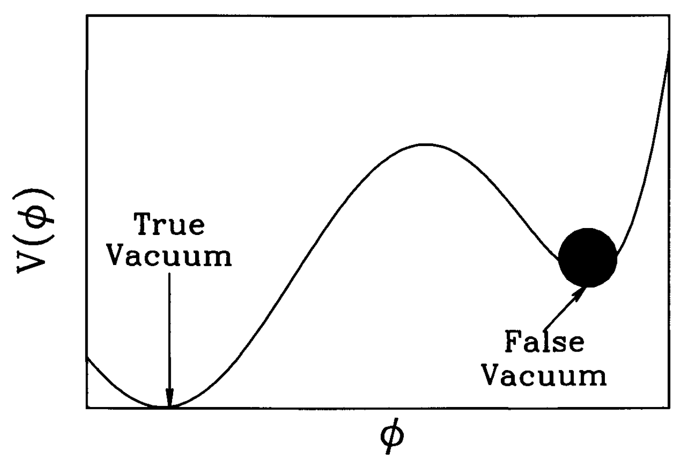

FIG. 2. A scalar field trapped in a false vacuum. Since it is trapped, it has little kinetic energy. The potential energy is nonzero, however, so the pressure is negative. The global minimum of the potential is called the true vacuum, since a homogeneous field sitting at the global minimum of the potential is in the ground state of the system. Figure from Dodelson (2003).

A. Old Inflation

The original idea for inflation by Guth in 1981 was that a scalar field could get trapped in a "false vacuum", essentially a local minimum in the potential function $V(\phi)$ at field position $\phi_{\rm FV}$. The picture is that the very early Universe was extremely hot, and the scalar field had a lot of kinetic energy and was bouncing around its potential. As the Universe expanded and cooled, the scalar could get trapped in the false vacuum state instead of relaxing to the true vacuum state (see Fig. 2). Essentially, the Universe gets *supercooled*, that is, it is left in a metastable state that has more energy than the "natural" lowest energy state.

When this happens, we have $\dot{\phi} \sim 0$, and if $V(\phi_{\rm FV}) > 0$, then the regions of the Universe where the field is trapped in the false vacuum will start inflating. These inflating regions will rapidly become exponentially larger than the region of the Universe in the true vacuum, and so start dominating the volume of the entire Universe. However, quantum mechanics tells us that there is nonzero probability for the scalar field to tunnel through the barrier to get to the true vacuum. Essentially, within the inflating region of false vacuum, bubbles of true vacuum will nucleate (a phase transition). Once a bubble has formed, its size expands at the speed of light. Inside the bubble, space is no longer inflating if $V = 0$ at the true vacuum. However, the space outside the bubble is still inflating and basically this process dilutes the true-vacuum bubbles faster then they can coalesce with each other. So inflation never really stops in this scenario: it has a "graceful exit" problem. This was realized very quickly after Guth's proposal, and this is why this scenario has been dubbed "old inflation". At nearly the same, Starobinsky in the Soviet Union proposed a model of inflation based on modified gravity, which to this day is still allowed by the data.

B. New/Chaotic Inflation

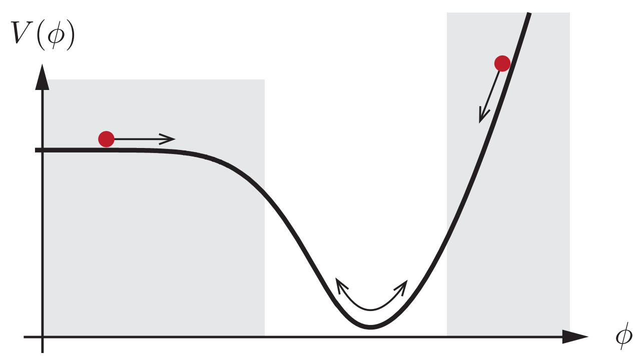

FIG. 3. Example of a slow-roll potential. Inflation occurs in the shaded parts of the potential. Figure from Baumann (2022).

A better model of inflation is gotten by taking a simple quadratic potential

$$V(\phi) = \frac{1}{2}m^2\phi^2, \tag{7}$$

where m is the mass of the scalar field. This model was proposed by Linde in 1982 in the Soviet Union, and a similar idea was brought forward by Albrecht and Steinhardt in the US. The nice thing about this potential is that, in Minkowski space, the equation of motion for the scalar field is

$$\ddot{\phi} + m^2\phi = 0, \tag{8}$$

that is, the equation of an harmonic oscillator. In this case we immediately know the dynamic of the scalar field: it will oscillate around the bottom of the potential with a frequency given by the mass m. Of course, in cosmology we are not in Minkowski spacetime but in an expanding FLRW Universe. It turns out that the expansion of the Universe turns the above equation into a *damped harmonic oscillator* equation

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \tag{9}$$

where H is the Hubble rate. This Hubble damping term is key for inflation since when $H \gg m$, the system is overdamped and the field get stuck somewhere on the potential at $V(\phi) > 0$. Neglecting the $m^2\phi$ term in this limit, the only physical solution to $\ddot{\phi} + 3H\dot{\phi} \simeq 0$ is $\phi =$ constant. Since the field is not moving moving much (the field is stuck) and is at $V(\phi) > 0$, inflation can start.

However, $\dot{\phi}$ is not *exactly* zero and the Hubble rate during inflation is not exactly constant, and in fact slowly decreases. At some point $H \sim m$ and the damping is no longer effective and the field can start oscillating around the minimum of the potential, at which point inflation stops. So this model does not suffer from a graceful exit problem. In addition, it contains a mechanism to *reheat* the Universe: as the field oscillates at the bottom of the potential, it can dump its energy to Standard Model particles.