PHYS480/581 Cosmology
Big-Bang Nucleosynthesis: Neutron Freeze-out

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I. THE STATE OF THE UNIVERSE AROUND $T \sim 1$ MEV

So far, we have seen how all the particles in the Standard Model of particle physics were in kinetic equilibrium with the hot cosmic plasma before annihilating away as the temperature of the bath dropped below their mass. By $T \gtrsim 1$ MeV, the only relativistic particles left were electrons, positrons, neutrinos, and photons in close thermal contact with each other. On the non-relativistic matter side, we had the baryons made of both protons and neutrons (there was also dark matter around, but since it doesn't really interact with anything except gravitationally, we will not discuss it further here). The abundance of these baryons was very small; this is usually characterized by the baryon asymmetry parameter $\eta_{\rm b}$, which is given by

$$\eta_{\rm b} \equiv \frac{n_{\rm b}}{n_\gamma} = 5.5 \times 10^{-10} \left(\frac{\Omega_{\rm b} h^2}{0.02}\right), \tag{1}$$

where $n_{\rm b}$ is the baryonic number density, and n_γ is the photon number density. This means that for every baryons, there are in average $\sim 10^{10}$ photons.

As we have seen, around $T \sim 1$ MeV the neutrinos decouple (i.e. stop exchanging energy and momentum) from the cosmic plasma and start free-streaming through the Universe. This occurs because neutrinos only interact via the weak interaction, which at $T \sim 1$ MeV is too "weak" to keep thermal equilibrium between neutrinos and the rest of the thermal bath. At the same time, another process is going out of equilibrium: the interconversion of neutrons and protons. This is no coincidence: proton and neutron conversion is also governed by the weak interaction. The key interconversion reactions are

$$p + \bar{\nu}_{\rm e} \leftrightarrow n + e^+, \qquad p + e^- \leftrightarrow n + \nu_{\rm e}, \qquad n \leftrightarrow p + e^- + \bar{\nu}_{\rm e}, \tag{2}$$

where p indicates a proton, and n a neutron. When these reactions become too slow compared to the expansion rate of the Universe, the comoving neutron abundance becomes "frozen out" (except for decays), and we thus call this process neutron "freeze-out". We care about this process because these neutrons form the reservoir from which other light nuclei will be assembled.

II. REVIEW OF NUCLEAR PHYSICS

A proton is an hydrogen nucleus ^{1}H (or just p), while a nucleus made of a neutron and a proton form deuterium, denoted ^{2}H or D. A nucleus formed of two neutrons and one proton is call tritium, ^{3}H or T. These are all isotopes of hydrogen; isotopes of a given element have the same number of protons but different number of neutrons. The superscript in front of the element denotes the *atomic mass number* A, which is the total number of protons and neutrons in the nucleus.

Nuclei with two protons are helium; these come in two variety ^{3}He (one neutron) and ^{4}He (two neutrons). In general, the mass of a nucleus is not equal to the sum of the masses of the protons and neutrons it contains. This is because some of that mass gets converted into nuclear *binding energy.* A nucleus of mass m with Z protons and $A - Z$ neutrons has a binding energy given by

$$B = Z m_{\rm p} + (A - Z) m_{\rm n} - m, \tag{3}$$

where $m_{\rm p} = 938.272$ MeV and $m_{\rm n} = 939.565$ MeV. For instance, ^{4}He has total binding energy of 28.3 MeV, while deuterium has binding energy of 2.2 MeV. To assess stability of nuclei, one is less interested in the total binding energy B, and more in the binding energy per nucleon, B/A. For ^{4}He, this is 7.07 MeV, while for deuterium this is 1.1 MeV, indicating that helium-4 is a much more stable nucleus than deuterium. Typical binding energies per nucleon are always of order $\sim$ MeV, and this why Big Bang nucleosynthesis (the synthesis of light nuclear elements in the early Universe) starts when the temperature has cooled just below that threshold.

Another important piece of physics that we will need moving forward is that *free* neutrons are unstable; they beta decay as $n \to p + e^- + \bar{\nu}_{\rm e}$ with a lifetime of $\tau_{\rm n} = 886.7 \pm 0.8$ sec. Once neutrons become bound in nuclei, they become stable. This means that all neutrons formed in the early Universe either decay away or become bound in light nuclei.

III. BIG BANG NUCLEOSYNTHESIS

In this class, we will not perform detailed calculations to determine the abundance of all light elements (H, D, T, ^{3}He, ^{4}He, etc.), which would require solving a large set of coupled differential equations. Instead, we will focus on determining the abundance of ^{4}He, which is by far the most abundant nuclei in the early Universe after hydrogen. This simpler problem neatly factorizes into two distinct parts: the freeze-out of the neutron abundance at temperatures just below 1 MeV, and the assembly of all the ^{4}He nuclei using the available neutrons.

A. Neutron Freeze-out

Before weak decoupling (when the weak interaction becomes too weak to maintain kinetic equilibrium) at $T \sim 1$ MeV, these reactions kept neutrons and protons in thermal equilibrium

$$p + \bar{\nu}_{\rm e} \leftrightarrow n + e^+, \qquad p + e^- \leftrightarrow n + \nu_{\rm e}. \tag{4}$$

Since the chemical potential of electrons and neutrinos was vanishingly small at these temperatures, these reactions set the chemical potential of neutrons and protons to be the same, $\mu_{\rm n} = \mu_{\rm p}$. In thermal equilibrium, the number density of protons and neutrons are given by

$$n_{\rm p,eq} = 2\left(\frac{m_{\rm p}T}{2\pi}\right)^{3/2} e^{-(m_{\rm p}-\mu_{\rm p})/T}, \qquad n_{\rm n,eq} = 2\left(\frac{m_{\rm n}T}{2\pi}\right)^{3/2} e^{-(m_{\rm n}-\mu_{\rm n})/T}, \tag{5}$$

respectively. Their ratio is then given by

$$\frac{n_{\rm p,eq}}{n_{\rm n,eq}} = \left(\frac{m_{\rm p}}{m_{\rm n}}\right)^{3/2} e^{(m_{\rm n}-m_{\rm p})/T}. \tag{6}$$

The prefactor is nearly unity (and thus can be neglected) since the proton and neutron mass are very close in value. However, this mass difference in the exponential is very important, and it is equal to $Q \equiv m_{\rm n} - m_{\rm p} = 1.293$ MeV. To a good approximation, this ratio can then be written as

$$\frac{n_{\rm p,eq}}{n_{\rm n,eq}} = e^{Q/T}. \tag{7}$$

For $T > Q$, this means that the abundance of neutrons and protons is the same. However, for $T < Q$ the abundance of neutrons become suppressed. If the weak interaction was able to maintain thermal equilibrium indefinitely, the neutron abundance would eventually drop to zero. But the weak interaction cannot maintain equilibrium, and the neutron abundance eventually freezes out. It is convenient to define the neutron fraction

$$X_n = \frac{n_{\rm n}}{n_{\rm n} + n_{\rm p}}, \tag{8}$$

which in equilibrium admits the simple expression

$$X_{n,\rm eq}(T) = \frac{e^{-Q/T}}{1 + e^{-Q/T}}. \tag{9}$$

Finding the abundance of neutrons after weak decoupling then requires determining the temperature at which this transition happens. This requires comparing the weak interaction rate $\Gamma_{\rm weak}(T)$ and the Hubble expansion rate $H(T)$ and finding the temperature T_f at which they are equal. A detailed calculation gives $T_f \simeq 0.8$ MeV, resulting in

$$X_{n,\rm eq}(T_f) \sim \frac{1}{6}. \tag{10}$$

In the following, we will use this simple estimate to compute the abundance of ^{4}He. A more detailed calculation yields $X_n(T_f) = 0.15$, which is pretty close to the above estimate.

B. Neutron Decay

The piece of physics that we are missing is neutron decay, but this is easy to take into account by an exponential factor

$$X_n(t) = X_{n,\mathrm{eq}}(T_f)e^{-t/\tau_n} \simeq \frac{1}{6}e^{-t/\tau_n} \tag{11}$$

where $\tau_\mathrm{n} = 886.7 \pm 0.8$ sec is the neutron lifetime. To compute the abundance of light nuclei, we thus need to know the time at which nucleosynthesis starts. With its large binding energy per nucleon, one might think that all available neutrons would rapidly assemble themselves into ^{4}He nuclei. However, for $T < 1$ MeV the matter density is just too low for three or more nucleons to come together and form heavier nuclei. Instead, helium production occurs first by assembling simpler nuclei in two-particle reactions, and then fusing these nuclei. The simplest such nucleus that first form is deuterium.