# PHYS480/581 Cosmology
# Cosmic Microwave Background

(Dated: November 9, 2022)

## I. PHOTON DECOUPLING

As we saw last time, hydrogen recombination starts in earnest at $z_{\rm rec} \simeq 1300$ and leads to a rapidly decreasing abundance of free electrons in the Universe. The process of hydrogen recombination is largely completed by $z \sim 700$, but a small but nonzero free electron abundance remains after that since hydrogen recombination proceeds out of equilibrium. Essentially, at some point it becomes very hard for the remaining free electrons to find the remaining free protons because of the expansion of the Universe. The ionization fraction freezes-out at around $X_e(z \ll z_{\rm rec}) \sim 6\times 10^{-4}$ at late times.

Before the onset of hydrogen recombination, the large number of photons present in the Universe constantly scatter off all the free electrons around via Thomson scattering (non-relativistic version of Compton scattering)

$$e^- + \gamma \leftrightarrow e^- + \gamma, \tag{1}$$

hence maintaining kinetic equilibrium between the photons and electrons. Kinetic equilibrium between protons and electrons is maintained via Coulomb scattering. As the number of available free electrons plummets, it becomes harder for photons to find free electrons to scatter on. This makes it difficult for the Thomson scattering process coupling photons to electrons to remain in equilibrium. The scattering rate for this process is

$$\Gamma_\gamma = \sigma_{\rm T} n_{\rm e}, \tag{2}$$

where

$$\sigma_{\rm T} = \frac{8\pi\alpha^2}{3m_{\rm e}^2} \tag{3}$$

is the Thomson cross section. Here, $\alpha = e^2/(4\pi\epsilon_0\hbar c)$ (in SI units, or just $\alpha = e^2/(4\pi)$ in natural units) is the electromagnetic fine-structure constant and $m_{\rm e}$ is the electron mass. The temperature at which photons stop scattering off free electrons can be estimated using the approximate criterion

$$\Gamma_\gamma(T_{\rm dec}) \sim H(T_{\rm dec}), \tag{4}$$

where $H$ is the Hubble expansion rate. Since we know that recombination begins on the matter dominated era of the Universe ($z_{\rm rec} < z_{\rm eq} \simeq 3400$), this means that photon decoupling, which occurs after recombination starts, will also happen during matter domination. Writing $n_{\rm e}(T) \simeq X_e(T) n_{\rm b}(T) = X_e(T)\eta_{\rm b} n_\gamma(T)$, we can write $\Gamma_\gamma(T)$ as

$$\Gamma_\gamma(T) = \frac{2\zeta(3)}{\pi^2}\eta_{\rm b}\sigma_{\rm T} X_e(T) T^3. \tag{5}$$

Meanwhile, the Hubble rate during matter domination (neglecting the small amount of remaining radiation) is

$$H(T) = H_0\sqrt{\Omega_{\rm m}}\left(\frac{T}{T_0}\right)^{3/2}, \tag{6}$$

where $T_0$ is the temperature of the photons today, and where we have used the fact that $T = T_0(1+z)$ to replace the redshift scaling in the Friedmann equation with a temperature scaling. Putting these two equations equal, we obtain

$$X_e(T_{\rm dec}) T_{\rm dec}^{3/2} \sim \frac{\pi^2}{2\zeta(3)}\frac{H_0\sqrt{\Omega_{\rm m}}}{\eta_{\rm b}\sigma_{\rm T} T_0^{3/2}}. \tag{7}$$

We now need to know what is $X_e(T_{\rm dec})$. We can use the Saha equation for $X_e$

$$\left(\frac{1-X_e}{X_e^2}\right)_{\rm eq} = \frac{2\zeta(3)}{\pi^2}\eta_{\rm b}\left(\frac{2\pi T}{m_{\rm e}}\right)^{3/2} e^{B_{\rm H}/T}, \tag{8}$$

to get an estimate for $X_e(T_{\rm dec})$. Since we expect that $X_e(T_{\rm dec}) \ll 1$, we have $1 - X_e(T_{\rm dec}) \approx 1$, and we can thus solve for $X_e(T_{\rm dec})$

$$X_e(T_{\rm dec}) \approx \left( \frac{2\zeta(3)}{\pi^2} \eta_{\rm b} \left( \frac{2\pi T_{\rm dec}}{m_{\rm e}} \right)^{3/2} e^{B_{\rm H}/T_{\rm dec}} \right)^{-1/2} \tag{9}$$

Using this we can write down

$$T_{\rm dec}^{3/4} e^{-B_{\rm H}/2T_{\rm dec}} \sim \frac{\pi^2}{2\zeta(3)} \frac{H_0\sqrt{\Omega_{\rm m}}}{\eta_{\rm b}\sigma_{\rm T} T_0^{3/2} \left( \frac{2\zeta(3)}{\pi^2} \eta_{\rm b} \left( \frac{2\pi}{m_{\rm e}} \right)^{3/2} \right)^{-1/2}} = C. \tag{10}$$

Taking the natural log on both sides leads to

$$T_{\rm dec} \sim \frac{B_{\rm H}}{2\left(\frac{3}{4}\ln\left(T_{\rm dec}/{\rm eV}\right) - \ln C\right)}. \tag{11}$$

This can be solved iteratively, starting with $T_{\rm dec} = 1$ eV (this sets the Log term to zero), which yields $T_{\rm dec} \simeq 0.25$ eV. Substituting this back in the Log, we can a converged answer of

$$T_{\rm dec} \simeq 0.26\,{\rm eV}, \tag{12}$$

which corresponds to a last-scattering redshift of

$$z_{\rm dec} \approx 1100. \tag{13}$$

This is the redshift at which the Universe becomes transparent to photons. This means that photons last-scatter around that redshift and then propagate freely through the Universe from that point on. These photons form today what we call the *cosmic microwave background* (CMB), since their wavelengths have been redshifted by the expansion to the microwave part of the electromagnetic spectrum. The age of the Universe when these photons are released is

$$t_{\rm dec} = \int_{z_{\rm dec}}^{\infty} \frac{dz}{(1+z)H(z)} \approx 380,000\,{\rm years}. \tag{14}$$

Thus, the CMB photons tell us about the state of the Universe as it stood a few hundred thousand years after the Big Bang. This is the earliest visible "picture" of the Universe that we will ever get, since before that the Universe was opaque. These CMB photons tell us that the Universe was largely homogeneous and isotropic at that time. However, it was not *exactly* homogeneous, and the CMB photons tell us that there were small (1 part in $10^5$) inhomogeneity at that time. It is thought that these early inhomogeneities eventually grow via gravitational infall to eventually form all the complex structures we observe in the Universe today.