*Pressure.**—Let me remind you where the $p^2/3E$ factor in (3.2.18) comes from. Consider a small area element of size $\mathrm{d}A$, with unit normal vector $\hat{\boldsymbol{n}}$ (see Fig. 3.3). All particles with velocity $|\boldsymbol{v}|$, striking this area element in the time interval between t and $t+\mathrm{d}t$, were located at $t=0$ in a spherical shell of radius $R=|\boldsymbol{v}|t$ and width $|\boldsymbol{v}|\mathrm{d}t$. A solid angle $\mathrm{d}\Omega^2$ of this shell defines the volume $\mathrm{d}V = R^2|\boldsymbol{v}|\mathrm{d}t\,\mathrm{d}\Omega^2$ (see the grey shaded region in Fig. 3.3). Multiplying the phase space density (3.2.14) by $\mathrm{d}V$ gives the number of particles in the volume (per unit volume in momentum space) with energy $E(|\boldsymbol{v}|)$,

$$\mathrm{d}N = \frac{g}{(2\pi)^3} f(E) \times R^2 |\boldsymbol{v}| \mathrm{d}t\,\mathrm{d}\Omega \,. \tag{3.2.19}$$

Not all particles in $\mathrm{d}V$ reach the target, only those with velocities directed to the area element. Taking into account the isotropy of the velocity distribution, we find that the total number of particles striking the area element $\mathrm{d}A\,\hat{\boldsymbol{n}}$ with velocity $\boldsymbol{v} = |\boldsymbol{v}|\,\hat{\boldsymbol{v}}$ is

$$\mathrm{d}N_A = \frac{|\hat{\boldsymbol{v}}\cdot\hat{\boldsymbol{n}}|\,\mathrm{d}A}{4\pi R^2} \times \mathrm{d}N = \frac{g}{(2\pi)^3} f(E) \times \frac{|\boldsymbol{v}\cdot\hat{\boldsymbol{n}}|}{4\pi}\,\mathrm{d}A\,\mathrm{d}t\,\mathrm{d}\Omega \,, \tag{3.2.20}$$

Taken from Baumann (2022)

where $\boldsymbol{v} \cdot \hat{\boldsymbol{n}} < 0$. If these particles are reflected elastically, each transfer momentum $2|\boldsymbol{p} \cdot \hat{\boldsymbol{n}}|$ to the target. Hence, the contribution of particles with velocity $|\boldsymbol{v}|$ to the pressure is

$$\mathrm{d}P(|\boldsymbol{v}|) = \int \frac{2|\boldsymbol{p} \cdot \hat{\boldsymbol{n}}|}{\mathrm{d}A\,\mathrm{d}t}\,\mathrm{d}N_A = \frac{g}{(2\pi)^3} f(E) \times \frac{p^2}{2\pi E} \int \cos^2\theta\, \sin\theta\, \mathrm{d}\theta\, \mathrm{d}\phi = \frac{g}{(2\pi)^3} \times f(E)\,\frac{p^2}{3E}\,, \quad (3.2.21)$$

where we have used $|\boldsymbol{v}| = |\boldsymbol{p}|/E$ and integrated over the hemisphere defined by $\hat{\boldsymbol{v}} \cdot \hat{\boldsymbol{n}} \equiv -\cos\theta < 0$ (i.e. integrating only over particles moving towards $\mathrm{d}A$—see Fig. 3.3). Integrating over energy E (or momentum p), we obtain (3.2.18).

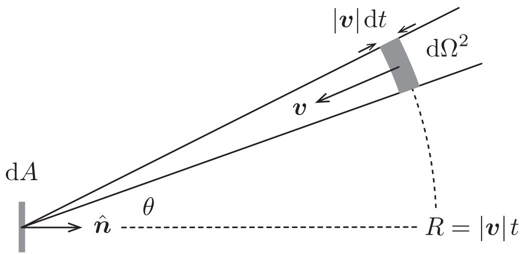

Figure 3.3: *Pressure in a weakly interacting gas of particles.*