PHYS480/581 Cosmology
Abundance of relativistic species in the Early Universe

(Dated: October 17, 2022)

I. EFFECTIVE NUMBER OF RELATIVISTIC SPECIES

At very early times in the history of the Universe ($T > 200$ GeV), a large number of known Standard Model particles were relativistic and thus contributing to the radiation budget of the Universe. Moreover, frequent interaction between all these particles through the electroweak or strong interactions kept them kinetic equilibrium at a common temperature T, leading to a radiation density

$$\rho_{\rm rad} = \sum_i \rho_i = \frac{\pi^2}{30} g_*(T) T^4, \tag{1}$$

where $g_*(T)$ is the *effective number of relativistic degrees of freedom* at temperature T. For particle in kinetic equilibrium with $T \gg m_i$, we simply have

$$g_*(T) = \sum_{i\in\text{bosons}} g_i + \frac{7}{8} \sum_{j\in\text{fermions}} g_j, \tag{2}$$

which is the sum over all the internal degrees of freedom of all relativistic particles present in the thermal bath. Sometimes, species could still be relativistic while being no longer in kinetic equilibrium with the rest of the cosmic plasma (this is relevant for neutrinos in our Universe), so a slightly more general expression for $g_*(T)$ is when we allow for each species to have their own temperature $T_i \gg m_i$,

$$g_*(T) = \sum_{i\in\text{bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{j\in\text{fermions}} g_j \left(\frac{T_j}{T}\right)^4. \tag{3}$$

In the above sum, we only count relativistic particles with $m_i \ll T$; once $T \lesssim m_i$, they are removed from the sum since they no longer contribute to the radiation density. Since the Standard Model (SM) of particle physics contains so many different particles with a broad range of masses, $g_*(T)$ is a strong function of temperature (or redshift). The detailed particle content of the SM is given in Fig. 1 below.

There are six families of quarks (each with their own antiparticle). These are spin-1/2 fermions, and each quark can have one of three colors, resulting in $6 \times 2 \times 2 \times 3 = 72$ quark (fermionic) degrees of freedom. The force carrier of the strong force (quantum chromodynamics, QCD) are massless spin-1 bosons (2 polarizations) and there are 8 different kind of gluons, for a total of $2 \times 8 = 16$ degrees of freedom. There are 3 families of charged leptons (with their respective antiparticle), each being a spin-1/2 particle. This leaves $3 \times 2 \times 2 = 12$ fermionic degrees of freedom. There are 3 families of neutrinos. However, only left-handed neutrinos and right-handed neutrinos exist in the SM (that is, not all helicity states are populated). This leaves $3 \times 2 = 6$ neutrino degrees of freedom. There are 3 weak massive gauge boson (spin-1), each with three polarizations, and one massless photon with two polarizations, leading to $3 \times 3 + 2 = 11$ bosonic degrees of freedom. Finally, we have the Higgs boson, a spin-0 particle with one degree of freedom. Thus, at $T \gtrsim 200$ GeV (i.e. above the top quark mass, the most massive particle in the SM), we have

$$g_*(T \gtrsim 200\,\text{GeV}) = 28 + \frac{7}{8} 90 = 106.75. \tag{4}$$

This of course assumes that there is not other particle beyond the SM, which is a big assumption. To determine the value of $g_*(T)$ at lower temperatures, we need to determine which particles has gone non-relativistic and annihilated away and remove their contributions from $g_*(T)$. Remember that for non-relativistic species in thermal equilibrium, their number density is $n \propto e^{-m/T}$, and most annihilations have occurred by $T \sim m/6$. For instance, the top quark annihilates first (for example through $t + \bar{t} \to \gamma + \gamma$), leading to $g_* = 106.75 - (7/8)12 = 96.25$ at temperatures near 30 GeV. $W^\pm$, Z^0 and H^0 annihilate next, leading to $g_* = 96.25 - (1 + 3 \times 3) = 86.25$ at $T \sim 10$ GeV. And so on for the b and c quarks, as well as the τ lepton. Fig. 2 shows the complete evolution of $g_*(T)$. At $T \sim 150$ MeV, something else happens: the QCD phase transition happens and remaining quarks and gluons form either 3-quark bound states (known as *baryons*, including protons and neutrons) or quark-antiquark bound states as *mesons* (e.g. pions). This represents a very sharp drops in $g_*(T)$ as all remaining quarks and gluons are now gone. However, the pions $\pi^\pm, \pi^0$ ($m_\pi \sim 135$ MeV) are still relativistic after the QCD transitions, so do contribute to g_* then.

type		mass	spin	g
quarks	$t, \bar{t}$	173 GeV	$\frac{1}{2}$	$2 \cdot 2 \cdot 3 = 12$
	$b, \bar{b}$	4 GeV		
	$c, \bar{c}$	1 GeV		
	$s, \bar{s}$	100 MeV		
	$d, \bar{s}$	5 MeV		
	$u, \bar{u}$	2 MeV		
gluons	g_i	0	1	$8 \cdot 2 = 16$
leptons	$\tau^\pm$	1777 MeV	$\frac{1}{2}$	$2 \cdot 2 = 4$
	$\mu^\pm$	106 MeV		
	$e^\pm$	511 keV		
	$\nu_\tau, \bar{\nu}_\tau$	< 0.6 eV	$\frac{1}{2}$	$2 \cdot 1 = 2$
	$\nu_\mu, \bar{\nu}_\mu$	< 0.6 eV		
	$\nu_e, \bar{\nu}_e$	< 0.6 eV		
gauge bosons	W^+	80 GeV	1	3
	W^-	80 GeV		
	Z^0	91 GeV		
	γ	0		2
Higgs boson	H^0	125 GeV	0	1

FIG. 1. Particle content of the Standard Model. Figure taken from Baumann (2022).

However, the pions and muons soon annihilate away at $T \sim 20$ MeV, leaving only the electrons, photons, and neutrinos (and their anti-particles) in the cosmic bath, leading to

$$g_*(10\,\mathrm{MeV}) = 2 + \frac{7}{8}(2 \times 2 + 3 + 3) = 10.75. \tag{5}$$

II. NEUTRINO DECOUPLING

After this, something happens that hasn't yet happened in the (known) thermal history of the Universe: some particles go out of kinetic equilibrium from the rest of the plasma while still relativistic. Up to this points, particles were in kinetic equilibrium (i.e. efficiently exchanging energy and momentum with the rest of the plasma) until they annihilated away. However, neutrinos can only interact with the rest of the plasma via the weak force. As the name says, this force is rather weak and not very good (compared to electromagnetic forces for instance) at maintaining kinetic equilibrium between neutrinos and the rest of the particles present (electrons, positrons, photons). Around $T \sim 1$ MeV, reactions like these can no longer maintain the neutrinos in kinetic equilibrium

$$e^+ + e^- \leftrightarrow \nu_e + \bar{\nu}_e \tag{6}$$
$$e^- + \bar{\nu}_e \leftrightarrow e^- + \bar{\nu}_e. \tag{7}$$

In cosmology, such breakdown of equilibrium always happens when the rate Γ of the process that was responsible for maintaining equilibrium falls below the Hubble expansion rate, that is,

$$\frac{\Gamma(T)}{H(T)} \lesssim 1. \tag{8}$$

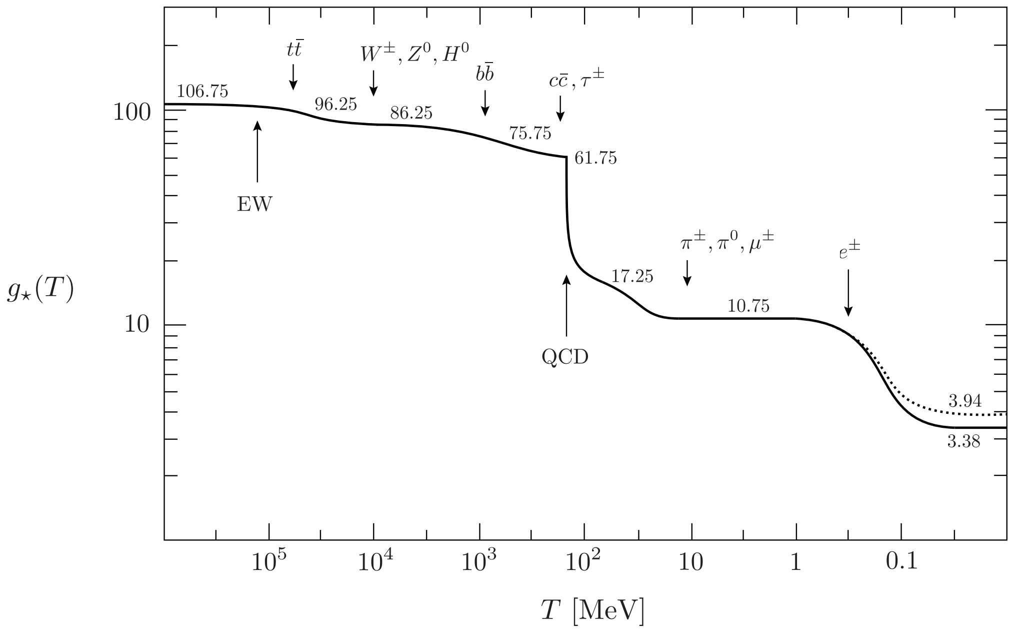

FIG. 2. Effective relativistic number of degrees of freedom in the early Universe as a function of temperature. Figure taken from Baumann (2022).

For neutrinos, this happens around $T \sim 1$ MeV. Below this temperature, neutrinos are no longer scattering with the rest of the plasma, and essentially become free-streaming particles propagating freely in the Universe. Without scattering, the shape of the neutrino particle distribution function $f_\nu(p)$ becomes frozen to what it was at decoupling: a Fermi-Dirac distribution for relativistic particles

$$f_\nu(p) = \frac{1}{e^{p/T_\nu} + 1}, \tag{9}$$

where T_ν is the neutrino temperature, which goes as $T_\nu \propto 1/a$ in an expanding Universe. This distribution function maintains this shape for the rest of the history of the Universe, even after neutrinos become non-relativistic.

For a little while after neutrino decoupling, nothing funny happens. We just have two separate sectors (one with just neutrinos, one with e^+, e^-, and γ) with the same temperature $T_\nu = T_\gamma$, both going as $1/a$ despite them not interacting. But a little while later, electron-positron annihilation occurs and they dump all their energy in the photon bath. This effectively reheats the photon bath relative to the neutrino bath, leading to $T_\gamma > T_\nu$ after e^+e^- annihilation. To compute this temperature difference, we need to discuss conservation of entropy in an expanding Universe, which we will do next time.