PHYS480/581 Cosmology
Overview and the Cosmological Principle

(Dated: August 21, 2022)

I. OVERVIEW OF COSMOLOGICAL HISTORY

Cosmology is the study of the Universe as a whole. Before discussing the fundamental assumptions underpinning our current cosmological model, it is a good idea to first visualize the different stages that our Universe has gone through during its evolution. Figure 1 shows a sketch of important milestones that our Universe achieved as it expanded and cooled down, focusing mostly on its later evolution. While this figure is not to scale in any possible sense of the word, it does illustrate that the bulk of the history of the Universe is spent forming galaxies, stars, and planets, with the onset of dark energy domination being a relatively recent phenomenon. Paradoxically, most of this course will *not* focus on this long era of galaxy and star formation, but rather on the eras preceding it (corresponding roughly to the first million years of the Universe). Back then, the Universe was much denser and hotter, and thus looked very different than it is today.

To better visualize these early eras of the Universe, Fig. 2 also shows a sketch of the evolution of the Universe but with a larger focus on what is happening at very early times. We see that as we go back in time, the study of the first million years of cosmological history requires first atomic physics, then nuclear physics, and finally particle physics. Note that anything happening earlier than the "Electroweak transition" is still rather speculative and should not be taken as absolute. While we will discuss in this course the inflationary epoch and the current epoch of accelerated expansion, the bulk of this course will be focused on the period following the "Electroweak transition" to roughly the era of recombination.

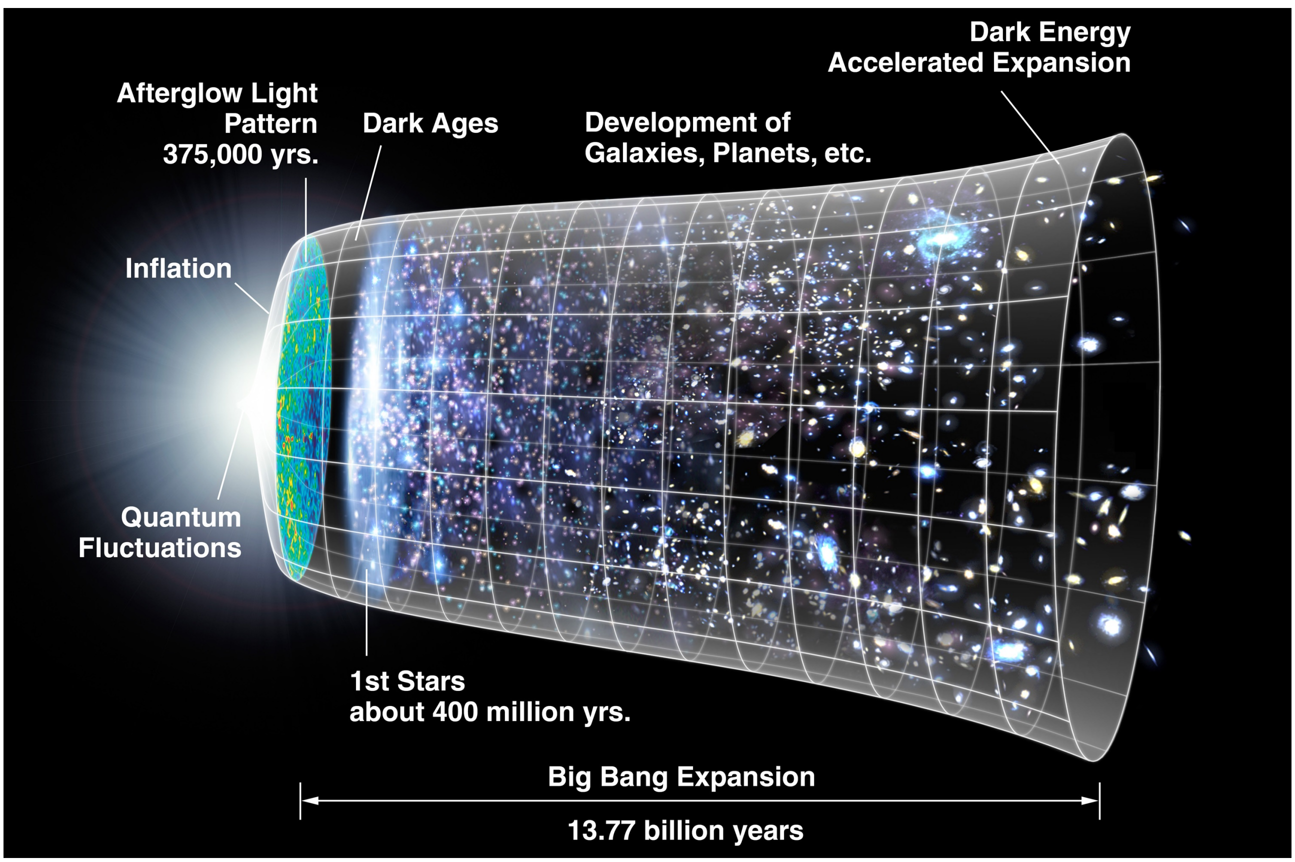

FIG. 1. Summary of the cosmic evolution of our Universe, with an emphasis on its later evolution. Time runs from left to right in this picture. Image credits: NASA/WMAP Science Team

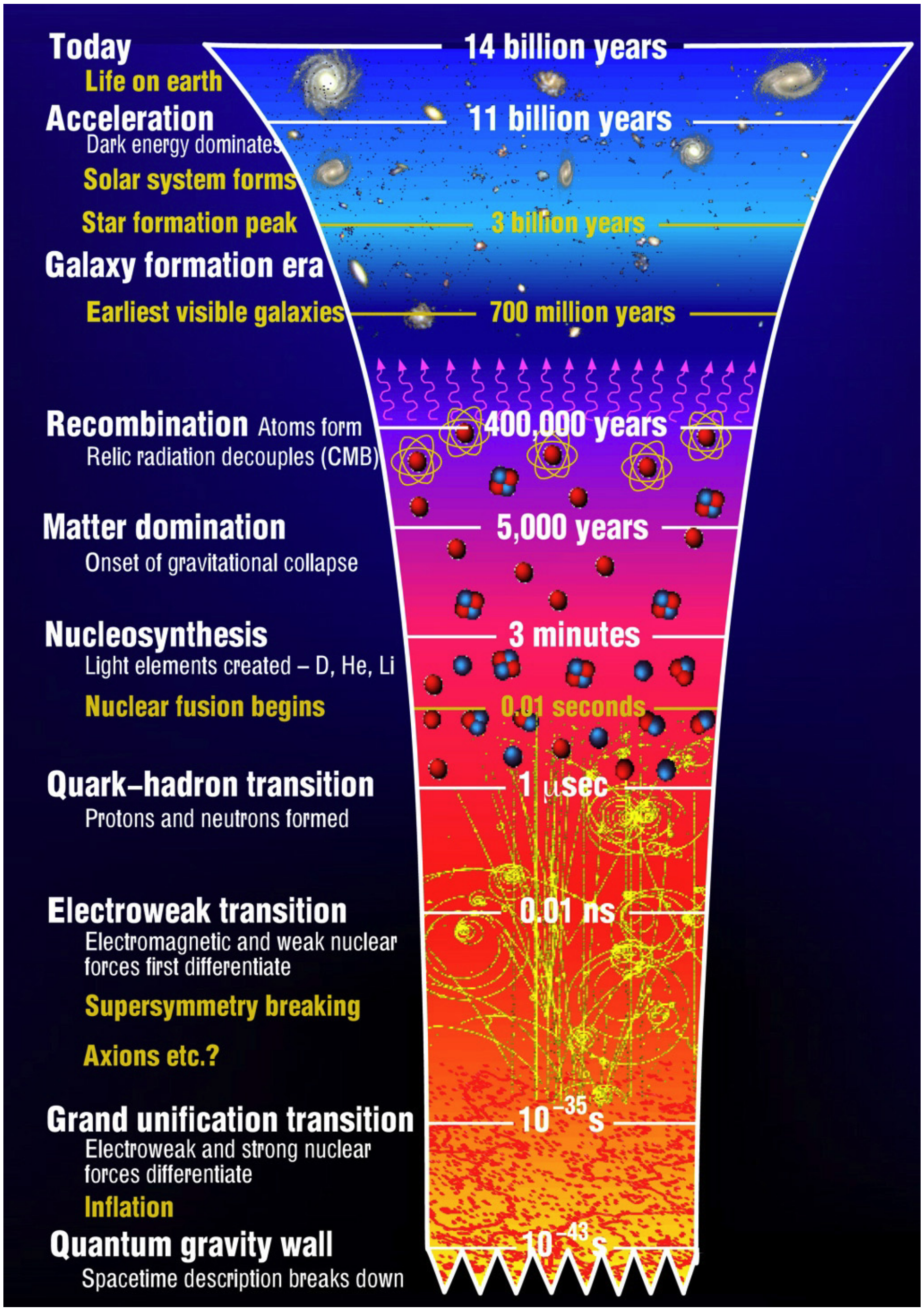

FIG. 2. Summary of the cosmic evolution of our Universe, with an emphasis on its early evolution. Time runs from bottom to top. Image credits: The Steven Hawking Centre for Theoretical Cosmology.

II. THE COSMOLOGICAL PRINCIPLE

The cosmological principle (sometime referred to as the Copernican principle) states that, on average, the Universe is homogeneous and isotropic. It also assumes that the laws of physics are the same everywhere.

- **Homogeneity** means that the Universe looks the same no matter where an observer is located. Of course, this is only strictly true when a large enough volume of the Universe is averaged over, as just choosing random points in the Universe could land you in the middle of a galaxy or in a void with nothing close by. The homogeneity of the Universe is thus really about comparing a large volume of the Universe situated here with another similar large volume situated on the other side of the Universe, and saying that the content of these two volumes are basically the same (see Fig. 3 for an illustration of this). More relevant to humanity, homogeneity says that there is no preferred location in the Universe. In particular, we are not at the center of the Universe. It is difficult to overstate the impact of this realization on how humans see their place in the cosmos.

Mathematically, homogeneity is related to the Universe having translation symmetry. Concretely, if you have a quantity $\rho(\mathbf{r}, t)$ (which could, e.g., represent the matter density at point $\mathbf{r}$ and time t), you can defined a *smoothed* version of this quantity over some length scale R as

$$\bar{\rho}_R(\mathbf{r}, t) = \int \rho(\mathbf{r}', t) W_R(\mathbf{r} - \mathbf{r}') d^3\mathbf{r}', \tag{1}$$

where $W_R(\mathbf{r})$ is a normalized smoothing kernel with typical length scale R. For example, $W_R(\mathbf{r})$ could be a Gaussian kernel. Note that the above is just a convolution. Homogeneity then states that, for sufficiently large R, we have

$$\bar{\rho}_R(\mathbf{r} + \mathbf{d}, t) = \bar{\rho}_R(\mathbf{r}, t), \tag{2}$$

for any arbitrary three-vector $\mathbf{d}$. As should be clear from the above, homogeneity is only true when comparing different patches of the Universe at the *same* cosmic time.

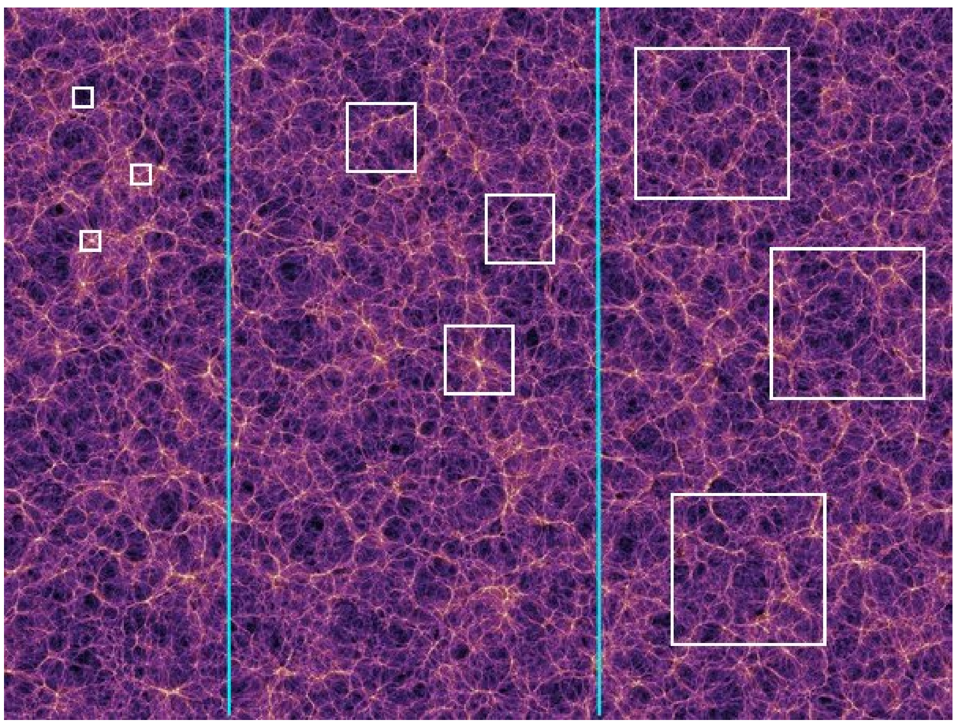

FIG. 3. N-body simulation of the large-scale structure of the Universe. The color scale shows the dark matter distribution in the late Universe, with lighter colors denoting regions of higher density. In the left column, the average density within each white square is very different, implying that the Universe is very inhomogeneous on these small scales. In the center column, the average density within each white square is similar but not the same, and the Universe is not quite homogeneous on these scales either. In the right column, the average density within each white square is the same and the Universe is said to be homogeneous on these (and larger) scales. Image credits: Millennium simulations.

- **Isotropy** states that the Universe looks the same in all directions. Again, this is only true in an average sense as should be familiar to you by looking at the night sky with the naked eye. In some directions, you see stars, planets, nebulae and galaxies, while in others you see nothing. Of course, we live a in a solar system, which itself lives in a galaxy, which itself lives in the Local Group. All these things *appear* to break isotropy locally by introducing special points on the sky (i.e. the position of the Sun, or of the center of the Milky Way). Here, we mean isotropy in a cosmological context, which means removing all these local structure to look at the broader distribution of objects (galaxies, quasars, etc.) on large cosmological scales. If you do so, the Universe indeed appears very isotropic. The point of isotropy is that there is no preferred direction in our Universe.

Mathematically, isotropy is related to the Universe having a rotation symmetry. Suppose we have a quantity $n(\theta, \phi)$, which might for example represent the number of observed galaxies by unit solid angle at a specific point (θ, ϕ) on the sky. We can define an *averaged* version of this quantity as

$$\bar{n}_\Omega(\theta, \phi) = \frac{1}{\Omega(\theta, \phi)} \int_{\Omega(\theta,\phi)} n(\theta', \phi') d\Omega', \tag{3}$$

where $\Omega(\theta, \phi)$ is a solid angle centered on the point (θ, ϕ), and $d\Omega' = \sin\theta' d\theta' d\phi'$ in the standard solid angle element. Isotropy then states that, for sufficient large solid angle Ω, we have

$$\bar{n}_\Omega(\theta + \theta_i, \phi + \phi_i) = \bar{n}_\Omega(\theta, \phi) \tag{4}$$

for any pair of angles (θ_i, ϕ_i). Perhaps the best observational test of isotropy is the cosmic microwave background (CMB); wherever you look at the sky, there is this 2.725 Kelvins microwave radiation coming at you. The uniformity of this radiation across the sky has played a major role in establishing our current cosmological model. But full disclosure, the CMB is only *nearly* isotropic; if you look in detail, you'll see fluctuations at the level of $\sim$1 part in 50,000 in its temperature (see Fig. 4). These perturbations are extremely important as it shows that the Universe was well on its way of forming structure about $380,000$ years after the Big Bang. We will further discuss the CMB later in the course.

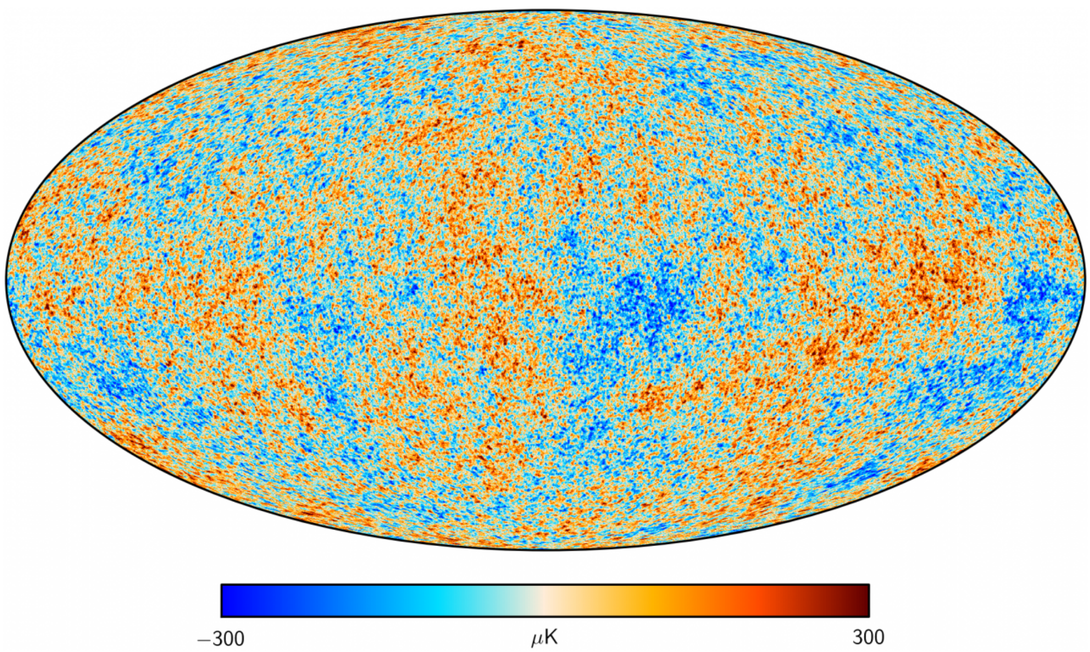

FIG. 4. CMB temperature anisotropies as imaged by the Planck satellite. Here, the mean 2.725K temperature has been subtracted in order to show the small fluctuations. Image credits: Planck Collaboration, ESA.

- The **Universality** of the laws of physics throughout the cosmos is something that is assumed *a priori* in cosmology. This is of course supported by observations (e.g. distant galaxies appear to behave similarly to nearby ones). It is also difficult to imagine that the Universe would be homogeneous if the laws of physics would be different from place to place. Nevertheless, theoretical physicists have proposed plenty of models where the laws of physics might have been different in the distant past.