PHYS480/581 Cosmology
Reheating after Inflation

(Dated: November 21, 2022)

I. BEHAVIOR OF SCALAR FIELD TOWARDS THE END OF INFLATION

As we discussed last time, one possible mechanism that could drive inflation is to have the energy density of the Universe dominated by the potential energy of a scalar field ϕ, often called the *inflaton* field. We gave the simplest possible example of such a potential

$$V(\phi) = \frac{1}{2}m^2\phi^2, \tag{1}$$

where the parameter m is known as the mass of the scalar field. The equation of motion for the field with this choice of potential is

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \tag{2}$$

which is the equation of a damped harmonic oscillator. As long as $H \gg m$, damping dominates and the field, if it started displaced from $\phi = 0$, is stuck somewhere high on the quadratic potential. This means that $\dot{\phi} \sim 0$ and $V(\phi) \sim$ constant and inflation can occur since the equation of state in this case is $w \approx -1$. Once $H \sim m$, the field can start moving more freely and begin oscillating about the minimum of the potential. Once $H \ll m$, one can forget about the Hubble friction term, and the equation of motion for the field is simply that of an harmonic oscillator

$$\ddot{\phi} + m^2\phi \approx 0, \tag{3}$$

which has for solution $\phi \sim \cos(mt)$ and $\phi \sim \sin(mt)$. For such harmonic motion, the pressure averaged over one oscillation cycle (period) vanishes

$$\langle P_\phi \rangle = \frac{1}{2}\langle \dot{\phi}^2 \rangle - \frac{1}{2}m^2\langle \phi^2 \rangle \approx 0, \tag{4}$$

since

$$\langle \phi^2 \rangle = \frac{\phi_0^2}{T}\int_0^T dt \cos^2(mt) = \frac{\phi_0^2}{2}, \tag{5}$$

and

$$\langle \dot{\phi}^2 \rangle = \frac{\phi_0^2 m^2}{T}\int_0^T dt \sin^2(mt) = \frac{\phi_0^2 m^2}{2}, \tag{6}$$

where the period is $T = 2\pi/m$, and ϕ_0 is the initial amplitude of the scalar field when it starts oscillating. Thus, an oscillating scalar field in an harmonic potential has zero pressure on average and thus behaves like non-relativistic *matter*. In the above, we assumed that the amplitude of oscillation stays constant. This is a pretty good approximation over one oscillation, but in the long run the amplitude of oscillation does decrease. We know this because non-relativistic matter always dilutes as $\rho_\phi \propto a^{-3}$, and we have

$$\langle \rho_\phi \rangle = \frac{1}{2}\langle \dot{\phi}^2 \rangle + \frac{1}{2}m^2\langle \phi^2 \rangle \approx \frac{1}{2}m^2\phi_0^2 \propto a^{-3}, \tag{7}$$

which implies that $\phi_0 \propto a^{-3/2}$.

II. REHEATING

While inflation ends when the inflaton field starts oscillating about the minimum of the potential (inflation thus ends "gracefully"), it doesn't immediately lead to a hot radiation-dominated Universe, which is necessary for Big

Bang Nucleosynthesis to happen. For this to happen, we need to allow for the inflaton field to *decay* to Standard Model particles. In practice, this modifies the fluid equation for the scalar field to

$$\dot{\rho}_\phi + 3H\rho_\phi = -\Gamma_\phi \rho_\phi, \tag{8}$$

where Γ_ϕ is the decay rate of the inflaton field. If $\Gamma_\phi \gg H$ and Γ_ϕ is approximately constant, the solution to the above equation is simply

$$\rho_\phi \propto e^{-\Gamma_\phi t}, \tag{9}$$

and the energy of the inflaton field is rapidly dumped into SM particles, hence reheating the Universe.