PHYS 480/581
Cosmology

Worksheet #11

Monday 10/03/2022

Question 1.

This problem is about the acceleration equation and the deceleration parameter.

(a) Using the Friedmann and fluid equations,

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \qquad \dot{\rho} + 3H(\rho + p) = 0, \tag{1}$$

Derive the acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{2}$$

(b) Show that the deceleration parameter

$$q_0 = -\frac{\ddot{a}(t_0)}{a(t_0)H_0^2} \tag{3}$$

for a universe with both matter ($\Omega_{\rm m}$) and dark energy (Ω_Λ) is given by

$$q_0 = \frac{\Omega_{\rm m}}{2} - \Omega_\Lambda. \tag{4}$$

Argue that for a spatially flat universe ($\Omega_K = 0$) with the above energy content, this implies that acceleration is only possible for $\Omega_\Lambda > 1/3$.