PHYS 480/581
Cosmology

Worksheet #15
Wednesday 10/26/2022

Question 1.

In this question, we want to determine the equilibrium abundance of deuterium nuclei, which are assembled via the reaction

$$n + p \leftrightarrow \mathrm{D} + \gamma. \tag{1}$$

(a) Assuming $\mu_\gamma = 0$ and that the above reaction is in chemical equilibrium, show that

$$\left(\frac{n_\mathrm{D}}{n_n n_p}\right)_\mathrm{eq} = \frac{3}{4}\left(\frac{m_\mathrm{D}}{m_n m_p}\frac{2\pi}{T}\right)^{3/2} e^{B_\mathrm{D}/T}, \tag{2}$$

where $B_\mathrm{D} = m_n + m_p - m_\mathrm{D} = 2.2$ MeV is the binding energy of deuterium. Note that deuterium is a spin-1 nucleus.

(b) Since $B_\mathrm{D} \ll m_p + m_n \simeq 1.9$ GeV, argue that the above can be approximately written as

$$\left(\frac{n_\mathrm{D}}{n_p}\right)_\mathrm{eq} = \frac{3}{4} n_{n,\mathrm{eq}}\left(\frac{4\pi}{m_p T}\right)^{3/2} e^{B_\mathrm{D}/T}. \tag{3}$$

(c) Using the approximation that $n_n \sim n_\mathrm{b}/2$, where n_b is the baryon number density, show that

$$\left(\frac{n_\mathrm{D}}{n_p}\right)_\mathrm{eq} \approx 4\eta_\mathrm{b}\left(\frac{T}{m_p}\right)^{3/2} e^{B_\mathrm{D}/T}, \tag{4}$$

where $\eta_\mathrm{b} \sim 6 \times 10^{-10}$ is the baryon to photon ratio. Thus, unless the exponential factor is large (which occurs for $T \ll B_\mathrm{D}$), the abundance of deuterium is very small.