PHYS 480/581
Cosmology

Worksheet #19
Monday 11/28/2022

Question 1.

Here, we want to show that dark matter fluctuations can grow logarithmically with the scale factor a during radiation domination. At these early times, the gravitational potential can be neglected and the evolution of matter fluctuations $\delta_{\rm m}$ is governed by the following equation

$$\ddot{\delta}_{\rm m} + 2H\dot{\delta}_{\rm m} \approx 0, \tag{1}$$

where $H = \dot{a}/a$ is the Hubble expansion rate.

(a) Show that if $\delta_{\rm m} = C \ln a$, then

$$\dot{\delta}_{\rm m} = CH, \qquad \text{and} \qquad \ddot{\delta}_{\rm m} = C\left(\frac{\ddot{a}}{a} - H^2\right), \tag{2}$$

where C is an arbitrary constant determined by the initial conditions.

(b) Now use the results from part (a), together with the acceleration and Friedmann equations during radiation domination,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho_{\rm rad} + 3P_{\rm rad}\right), \qquad \text{and} \qquad H^2 = \frac{8\pi G}{3}\rho_{\rm rad}, \tag{3}$$

to show that $\delta_{\rm m} = C \ln a$ is indeed a solution to Eq. (1). Remember to use the radiation equation of state to relate $P_{\rm rad}$ to $\rho_{\rm rad}$.

This thus shows that cold dark matter fluctuations can start growing even while the Universe is radiation dominated.