PHYS 480/581
Cosmology

Worksheet #20

Wednesday 11/30/2022

Question 1.

Here, we want to show that dark matter fluctuations can grow proportionally to the scale factor a during matter domination, $\delta_{\rm m} \propto a$. The equation governing the evolution of such fluctuations is

$$\ddot{\delta}_{\rm m} + 2H\dot{\delta}_{\rm m} - 4\pi G\bar{\rho}_{\rm m}\delta_{\rm m} = 0, \tag{1}$$

where $H = \dot{a}/a$ is the Hubble expansion rate, and $\bar{\rho}_{\rm m}$ is the mean dark matter density.

(a) Show that if $\delta_{\rm m} = Ca$ (where C is an arbitrary constant), then the above equation can be written as

$$\frac{\ddot{a}}{a} + 2H^2 - 4\pi G\bar{\rho}_{\rm m} = 0. \tag{2}$$

(b) Now use the acceleration and Friedmann equations during matter domination,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\bar{\rho}_{\rm m} + 3P_{\rm m}\right), \qquad \text{and} \qquad H^2 = \frac{8\pi G}{3}\bar{\rho}_{\rm m}, \tag{3}$$

to show that $\delta_{\rm m} = Ca$ is indeed a solution to Eq. (1). Remember to use the matter equation of state to determine $P_{\rm m}$.

This thus shows that cold dark matter fluctuations can grow rapidly during matter domination