PHYS 480/581
Cosmology

Homework Assignment 2

Due date: Wednesday October 5 2022, in class

Question 1 (2 points).

Show that for close-by objects at a distance d away from us ($H_0 d \ll 1$), the redshift z is approximately given by

$$z \simeq H_0 d. \tag{1}$$

Does it matter whether d is a comoving, angular diameter, or luminosity distance?

Question 2 (3 points).

A galaxy emits light of a particular wavelength. As the light travels, the expansion of the Universe slows down and stops. Just after the Universe begins to recollapse, the light is received by an observer in another galaxy. Does the observer see the light redshifted or blueshifted?

Question 3 (4 points).

In a flat spacetime, objects of a fixed physical size subtend smaller and smaller angles as they are further and further away; in an expanding universe this is not necessarily true. Consider the angular size $\theta(z)$ of an object of physical size L at redshift z. In a spatially-flat universe with $\Omega_{\rm m} = 0.3$ and $\Omega_\Lambda = 0.7$, at what redshift is $\theta(z)/L$ minimum? Assuming $H_0 = 70$ km/s/Mpc, what is the angular size of a galaxy of physical size 10 kiloparsecs at this redshift? What is the angular size of this galaxy at redshift $z = 10$?

Question 4 (5 points).

For this question, consider a flat FLRW universe with $H_0 = 67.66$ km/s/Mpc, $\Omega_{\rm m} = 0.311$, $\Omega_{\rm rad} = 9.1 \times 10^{-5}$, and $\Omega_\Lambda = 1 - \Omega_{\rm m} - \Omega_{\rm rad}$.

(a) Plot the age of the Universe as a function of redshift. Remember that $z = 0$ corresponds to the present age of the Universe.

(b) Starting at the Big Bang ($z = \infty$), what fraction of the current age of the Universe has elapsed by $z = 2$? What about $z = 10$?

(c) How old was the Universe at $z = 1090$, when the photons making up the cosmic microwave background were released?

Question 5 (10 points).

Observations of Type Ia supernovae were instrumental in the discovery of the acceleration of the expansion and the related presence of dark energy. In this question, we will use some recent supernova data to indeed visually show that a flat universe with $\Omega_\Lambda = 0.7$ and $\Omega_\mathrm{m} = 0.3$ is a much better fit to the data than a matter-dominated universe with $\Omega_\mathrm{m} = 1$. Start by downloading the data here (a link is also posted on the course webpage). The file contains three columns, giving the redshift (z), the apparent magnitude of the supernova m, and the error on the apparent magnitude Δm, respectively.

(a) Using your favorite plotting package, plot m versus z for all supernovae in the data file. Make sure to include the error bar on m for each data point. Clearly label your axes.

(b) Now add the predictions for the two cosmological models mentioned above (Model 1: $\Omega_\Lambda = 0.7, \Omega_\mathrm{m} = 0.3$; Model 2: $\Omega_\mathrm{m} = 1$) on your plot. To do so, remember that the predicted apparent magnitude $m_\mathrm{pred}(z)$ for a supernova at redshift z is related to the absolute supernova magnitude M and the distance modulus $\mu(z)$ by

$$m_\mathrm{pred}(z) = \mu(z) + M, \tag{2}$$

where $M = -19.4$ and

$$\mu(z) = 5\log_{10}\left(\frac{d_\mathrm{L}(z)}{\mathrm{Mpc}}\right) + 25, \tag{3}$$

where $d_\mathrm{L}(z)$ is the luminosity distance to redshift z in Mpc. Use $H_0 = 67.5$ km/s/Mpc and be mindful of the units when computing d_L. Make sure the two models and the data points (with their error bars) are clearly visible on your plot.

(c) Which model appears to be a better fit to the data? Is this conclusion robust to changing M?

Question 6 (4 points).

For this question, assumes the same cosmological parameters given in question 4 above, unless otherwise mentioned.

(a) Compute the redshift at which the energy density in dark energy is equal to that of matter. How much time has elapsed between that epoch and today? Compare this timescale to the age of the Solar system (4.6 Gyrs). This is often referred to as the "coincidence" problem.

(b) Imagine a universe in which there are 2 extra species of massless neutrinos (in addition to those present in the Standard Model of particle physics). What is the redshift of matter-radiation equality in such a universe? Assume that the extra neutrinos have the same temperature as the standard ones.