PHYS 480/581 - Solutions of Homework 5

Q1

Using Eq. 7 from the lecture notes of photon decoupling with $X_e = 1$ (completely ionized), we get the decoupling temperature

$$T_{dec} \simeq \left(\frac{\pi^2}{2\xi(3)} \frac{H_0\sqrt{\Omega_m}}{\eta_b \sigma_T T_0^{3/2}} \right)^{2/3} \simeq 0.009 \ \ eV. \tag{1}$$

We used the numerical values given in the question and the known fact that $T_0 = 2.35 \times 10^{-4}$ eV. The corresponding redshift is

$$z_{dec} = \frac{T_{dec}}{T_0} - 1 \simeq 37.3 \tag{2}$$

Thus, without recombination, photons would decouple much later than the normal case.

Q2

a)

We knew that the total number of relativistic degrees of freedom of the Standard Model is $g_* = 106.75$. At GUT scale, all particles are relativistic. So, we get

$$H_I = \sqrt{\frac{\pi^2}{90 M_p^2} g_*(T_{GUT}) T_{GUT}^4} \simeq 1.4 \times 10^{12} GeV, \tag{3}$$

where $M_P \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck mass, $T_{GUT} \simeq 10^{15}$ GeV.

Now we do a unit conversion

$$H_I^{-1} = \frac{\hbar c}{1.4 \times 10^{12} GeV} \simeq 1.4 \times 10^{-28} m, \tag{4}$$

where $\hbar \simeq 6.58 \times 10^{-25}$ Gev.s and $c = 3 \times 10^8$ m/s.

b)

Due to the scaling law $T \propto 1/a$, we have

$$T_{end} = T_0 \frac{a_0}{a_{end}} \Rightarrow \frac{a_0}{a_{end}} = \frac{T_{end}}{T_0} = \frac{10^{24} eV}{2.348 \times 10^{-4} eV} \simeq 4 \times 10^{27}. \tag{5}$$

The subscripts "end" indicate the moment of the end of inflation. $T_{end} \simeq T_{GUT} \simeq 10^{15} GeV = 10^{24} eV$.

c)

At the end of inflation, we have

$$H_{end}^{-1} = e^N H_I^{-1}. \tag{6}$$

But we also have

$$\frac{3.1 H_0^{-1}}{H_{end}^{-1}} = \frac{a_0}{a_{end}}. \tag{7}$$

A way to think about this is that: *physical distance* = *comoving distance* × *scale factor*, but the comoving distance is fixed so it does not appear in our ratio. From these two equations, we get

$$N = \ln\left(\frac{a_{end}}{a_0}\frac{3.1 H_0^{-1}}{H_I^{-1}}\right) \simeq 62. \tag{8}$$

We of course used the result of part (b) and the given values of H_0 and H_I.

d)

For the flatness problem, we just do exactly the same thing we did in part (c) except that now we have a bound

$$N > \ln\left(\frac{a_{end}}{a_0}\frac{10^3 H_0^{-1}}{H_I^{-1}}\right) \simeq 68. \tag{9}$$

So, we see that the flatness problem asks for a stronger condition than the horizon problem.

Q3

Assuming uniform distribution of dark matter halo, the distance d from the galactic center to the nearest black hole would be inferred from

$$10^{-10} M_\odot \simeq \frac{10^{12} M_\odot}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi d^3 \tag{10}$$

$$\Rightarrow d \simeq \left[10^{-22}(200\ \ kpc)^3\right]^{1/3} \approx 9.28 \times 10^{-6} kpc. \tag{11}$$

From Google, the diameter of the Solar System is around 2.92×10^{-7} kpc. Thus, we see that the distance from the galactic center to the nearest black hole is roughly ten times the size of the Solar System.