

PHYS 480/581 General Relativity

Extra Problems #2

Question 1.

In the following $\Lambda^\beta{}_\nu$ is a Lorentz transformation, $\eta_{\mu\nu}$ is the Minkowski metric, and $\delta^\mu{}_\nu$ is the Kronecker delta.

- (a) Show that $(\Lambda^{-1})^\alpha{}_\mu \eta_{\alpha\nu} = \eta_{\mu\beta} \Lambda^\beta{}_\nu$
- (b) Compute $\delta^\mu{}_\mu$.

Question 2.

Here we consider the electromagnetic field strength tensor $F^{\mu\nu}$, which is antisymmetric in its two indices, that is, $F^{\mu\nu} = -F^{\nu\mu}$.

- (a) Show that

$$\eta_{\mu\nu} F^{\mu\nu} = 0. \quad (1)$$

- (b) In relativistic component notation, the electromagnetic Lorentz force law for a particle with electric charge q is

$$\frac{dp^\mu}{d\tau} = q F^{\mu\nu} \eta_{\nu\alpha} u^\alpha, \quad (2)$$

where p^μ is the four-momentum, τ is proper time, and u^α is the four-velocity. This equation implies that a charged particle moving in an electromagnetic field generally has a changing four-momentum (which matches our physical intuition). Show that this time-dependent four-momentum is nonetheless always consistent with the standard relation

$$\frac{d(p_\mu p^\mu)}{d\tau} = 0, \quad (3)$$

which follows from the normalization of the four-momentum $p_\mu p^\mu = -m^2 = \text{constant}$, where m is the mass of the particle.