

PHYS 480/581 General Relativity

Extra Problems #4

Question 1.

The metric for a two-sphere is

$$ds^2 = a^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where a the the radius of the sphere. This metric tells you how to compute distances on the surface of a sphere of constant radius. Now, imagine that there is a rank-4 tensor $\mathbf{R}$ living on this two-sphere, with the following properties

$$R_{\rho\sigma\mu\nu} = -R_{\sigma\rho\mu\nu}, \quad R_{\rho\sigma\mu\nu} = -R_{\rho\sigma\nu\mu}, \quad (2)$$

and

$$R_{\rho\sigma\mu\nu} = R_{\mu\nu\rho\sigma}. \quad (3)$$

where here $\rho, \sigma, \mu, \nu = \theta$ or ϕ . Say I give you one component of this tensor

$$R^\theta_{\phi\theta\phi} = \sin^2 \theta. \quad (4)$$

- (a) Using the properties given above and Eq. (4), find all 16 components of $\mathbf{R}$ with all four lower indices, i.e., $R_{\rho\sigma\mu\nu}$.
- (b) Define the second-rank tensor $\tilde{\mathbf{R}}$ with components given by

$$\tilde{R}_{\mu\nu} = R^\alpha_{\mu\alpha\nu}. \quad (5)$$

Find the four components of $\tilde{\mathbf{R}}$.

- (c) Find the trace of $\tilde{\mathbf{R}}$, $\tilde{R}^\mu_\mu$.