

PHYS 480/581 General Relativity

Extra Problems #7

Question 1.

Consider the Poincaré half-plane, which has for metric

$$ds^2 = \frac{a^2}{y^2}(dx^2 + dy^2), \tag{1}$$

with $y > 0$, and where a is a constant.

- (a) Compute the length of a $x = \text{constant}$ line segment between the coordinates y_1 and y_2 , with $y_2 > y_1$. Could an observer reach $y = 0$ by traveling a finite distance.
- (b) Show that the geodesics in this space are either semi-circles with centers located on the x -axis or $x = \text{constant}$ lines.
- (c) Compute the one independent component of the Riemman tensor. Show that the scalar curvature (Ricci scalar) of this manifold is the same everywhere.
- (d) In 2D, manifolds that have $R = \text{constant}$ are said to *maximally symmetric*. How many symmetries does the above metric have? Use inspiration from the Euclidean space $\mathbb{R}^2$.