

PHYS 480/581 General Relativity

Worksheet #1
Monday August 31 2026

Question 1.

Consider the Lorentz transformation for a boost in the positive z -direction,

$$\Lambda^\mu{}_\nu = \begin{bmatrix} \gamma & 0 & 0 & -\gamma\beta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\gamma\beta & 0 & 0 & \gamma \end{bmatrix}, \quad (1)$$

where $\gamma = 1/\sqrt{1 - \beta^2}$. Also consider a vector $\mathbf{A}$ with the following components $A^\mu = (V, 0, 0, 0)$ in flat Minkowski spacetime with $g_{\mu\nu} = \eta_{\mu\nu}$.

- (a) Determine the components of $\Lambda_{\mu\nu}$.
- (b) What are the components of the dual vector A_μ ?
- (c) Determine the components of $\mathbf{A}$ in the boosted frame, that is, $A'^\mu = \Lambda^\mu{}_\nu A^\nu$. What about A'_μ ?

Question 2.

For an arbitrary vector $\mathbf{A}$ in Minkowski spacetime, show that

$$\frac{d}{d\tau}(\mathbf{A}^2) = 2\eta_{\mu\nu}A^\mu \frac{dA^\nu}{d\tau}. \quad (2)$$