PHYS 480/581
General Relativity

Worksheet #2

Wednesday September 2 2026

Question 1.

Consider a 2D parabolic coordinate system S' with coordinates p', q', which are related to the ordinary cartesian coordinates x, y by the transformations

$$p'(x,y) = x, \qquad q'(x,y) = y - cx^2, \tag{1}$$

where c is a constant.

(a) Derive the inverse transformation between the two coordinate systems, that is, $x(p', q')$ and $y(p', q')$.

(b) Calculate all eight partial derivatives $\partial x'^{\mu}/\partial x^{\nu}$ and $\partial x^{\mu}/\partial x'^{\nu}$.

(c) If the metric in the cartesian coordinate system x, y is simply Euclidean with

$$g_{\alpha\beta} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \tag{2}$$

use the transformation law for the metric

$$g'_{\mu\nu} = \frac{\partial x^{\alpha}}{\partial x'^{\mu}} \frac{\partial x^{\beta}}{\partial x'^{\nu}} g_{\alpha\beta} \tag{3}$$

to show that the metric in the p', q' coordinate system takes the form

$$g'_{\mu\nu} = \begin{bmatrix} 1 + 4c^2 p'^2 & 2cp' \\ 2cp' & 1 \end{bmatrix}. \tag{4}$$

Note that the above metric is not diagonal and that 3 of its entries are not constant. This metric nonetheless represents standard 2D Euclidean space, just written in a strange coordinate system.

(d) Consider a vector $\mathbf{A}$, which in the p', q' coordinate system has components $A'^{\mu} = (1, 0)$. Find the components of $\mathbf{A}$ in the x, y coordinate system. Verify that $A_{\mu}A^{\mu} = A'_{\mu}A'^{\mu}$.