PHYS 480/581
General Relativity

Worksheet #3

Wednesday September 9 2026

Question 1.

(a) Consider standard 3D Euclidean space in cartesian coordinates (x, y, z). In this coordinate system, the metric is just the identity

$$g_{ij} = \delta_{ij}. \tag{1}$$

Now consider a coordinate transformation into spherical coordinates (r', θ', ϕ')

$$x = r' \cos\phi' \sin\theta', \quad y = r' \sin\phi' \sin\theta', \quad z = r' \cos\theta', \tag{2}$$

$$r' = \sqrt{x^2 + y^2 + z^2}, \quad \theta' = \cos^{-1}\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right), \quad \phi' = \tan^{-1}\left(\frac{y}{x}\right). \tag{3}$$

Derive the metric g'_{kl} in this spherical coordinate system. Use this to write the squared line element ds^2.

(b) Compute the inverse metric g'^{kl}.

Question 2.

Consider the following $(1, 2)$ tensor $C^{\alpha}{}_{\beta\mu}$. This object is a tensor since it transforms as

$$C'^{\gamma}{}_{\delta\nu} = \frac{\partial x'^{\gamma}}{\partial x^{\alpha}} \frac{\partial x^{\beta}}{\partial x'^{\delta}} \frac{\partial x^{\mu}}{\partial x'^{\nu}} C^{\alpha}{}_{\beta\mu} \tag{4}$$

under a $x \to x'(x)$ coordinate transformation.

(a) Show that the contraction $A_\mu \equiv C^{\alpha}{}_{\alpha\mu}$ is indeed a dual vector.

(b) Show that $C_{\alpha\beta\mu}$ is a (0,3) tensor.